Calculating the forced response of the tropospheric circulation

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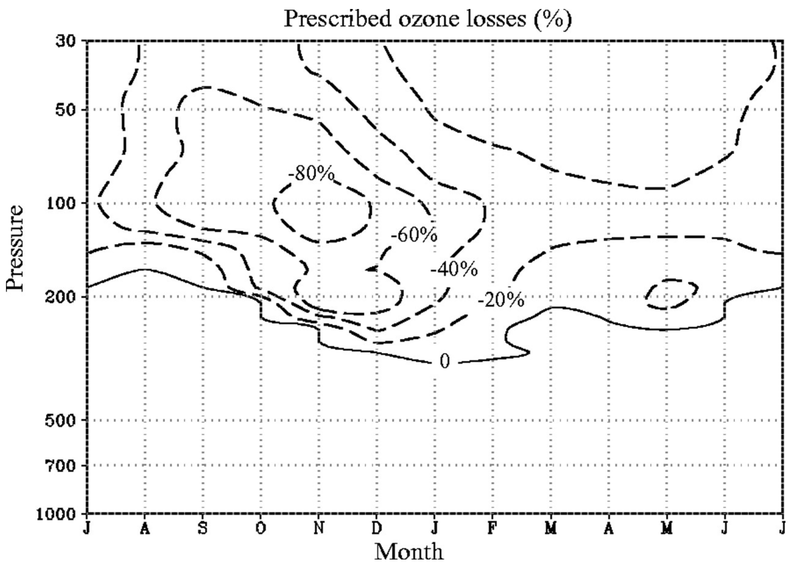
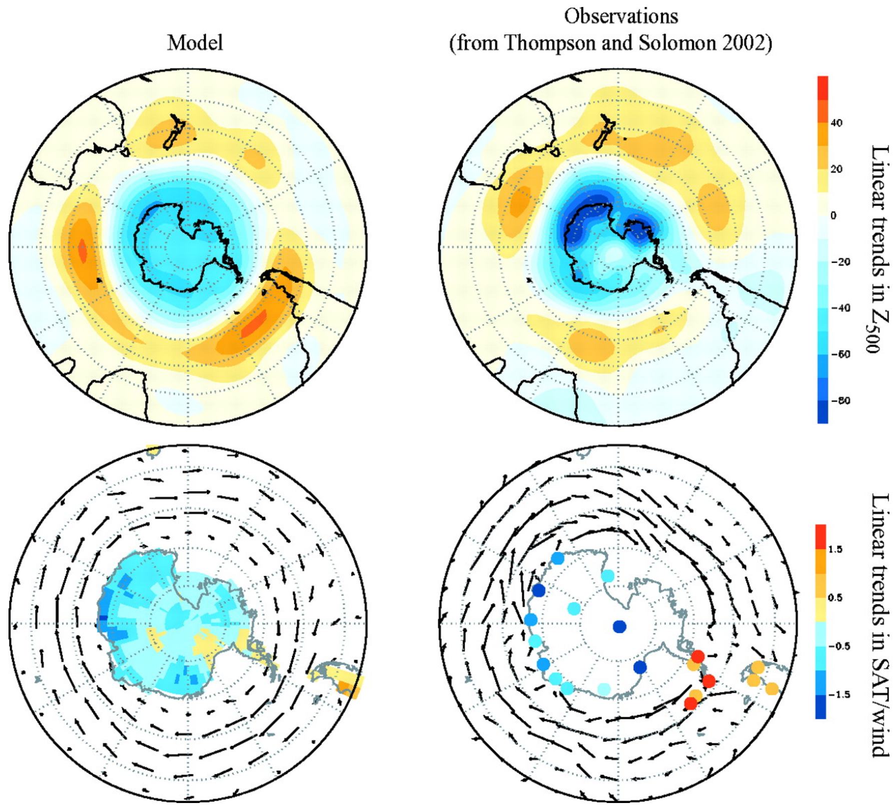
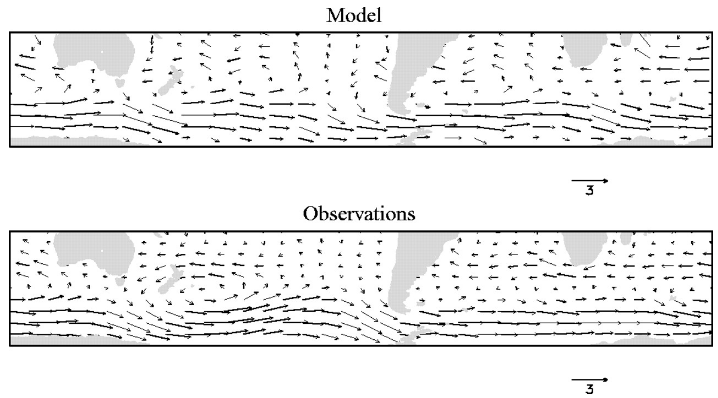
work reported in CH2011 JAS (and possible sequels)

Changes in the SH troposphere as a dynamical response to stratospheric ozone depletion



Ozone perturbation applied to AGCM

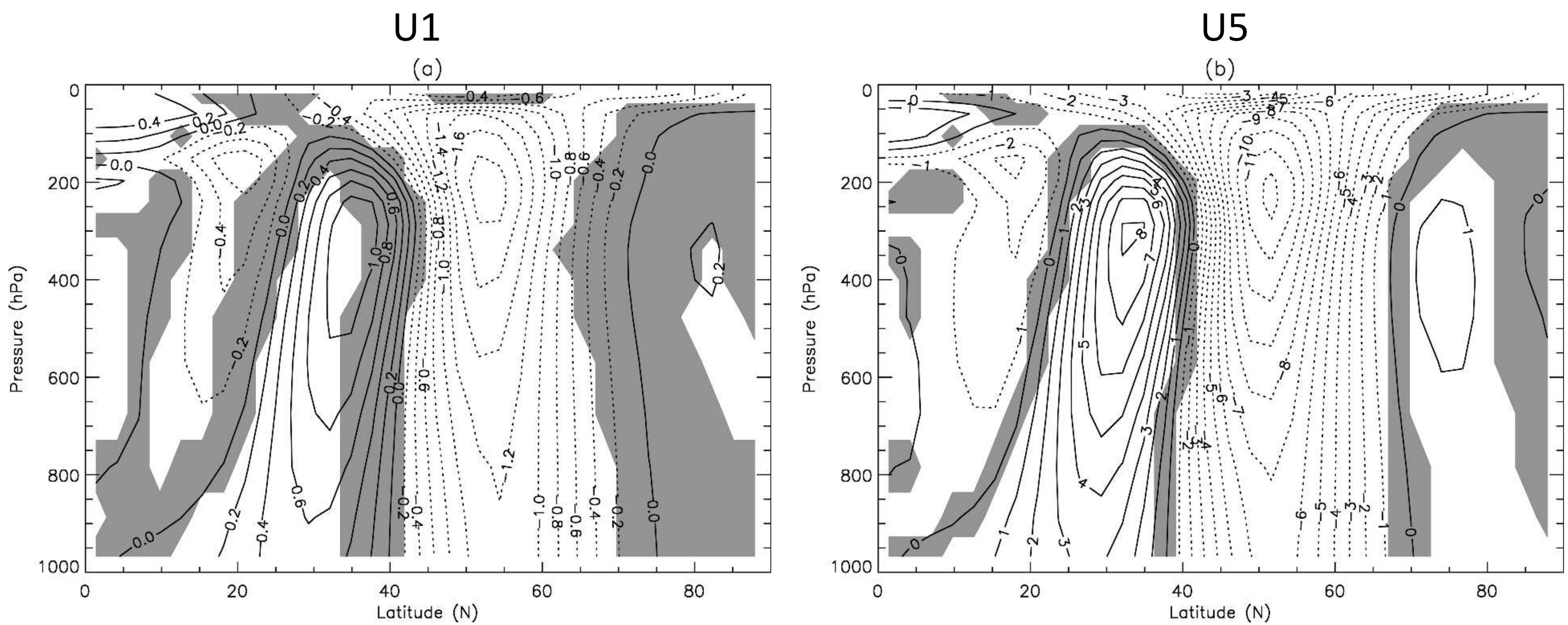




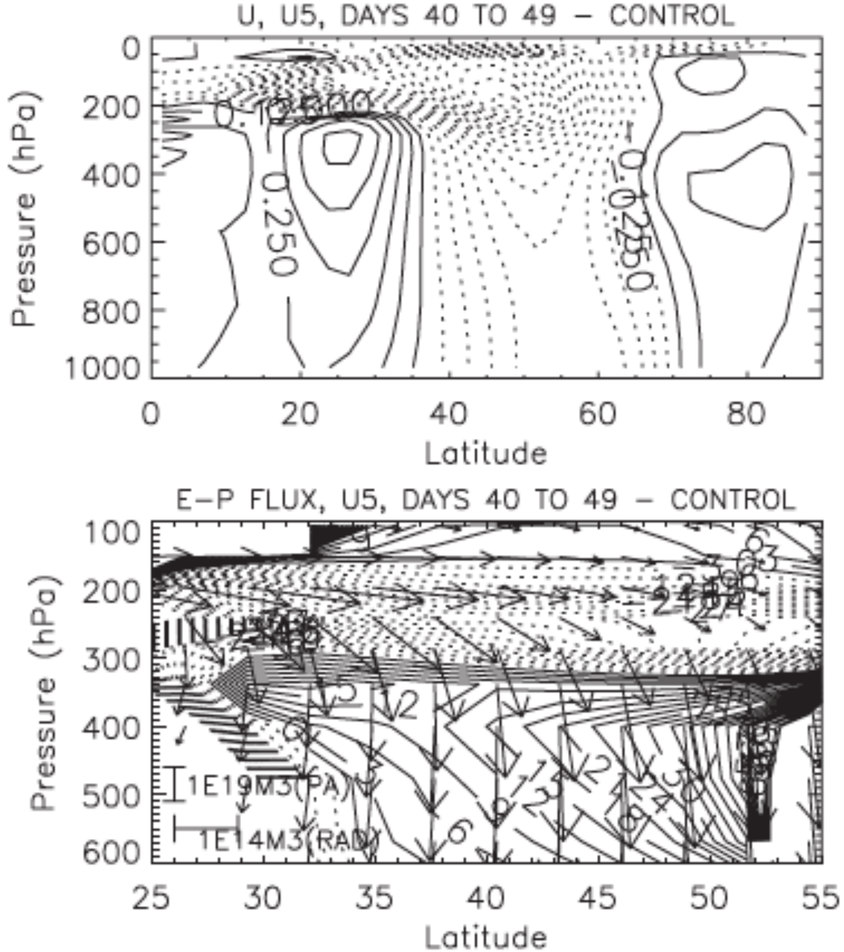
(Thompson and Solomon 2002, Gillett and Thompson, 2003)

Haigh et al (2005)

Response of simple troposphere to imposed changes, e.g. uniform increase in radiative equilibrium temperature in stratosphere



Simpson et al (2009)



We know that changes In eddy fluxes are a vital part of the tropospheric dynamical response – possibility of 'amplification'

Simpson et al (2009) analyse in terms of index of refraction – multi-stage adjustment – induced changes in u affect changes in index of refraction and hence eddy fluxes etc.

Response to 'solar cycle' forcing is one example of more general response problem: ozone hole (Gillett and Thompson 2003), stratospheric perturbation (Polvani and Kushner 2002, Song and Robinson 2004), surface friction (Chen et al 2007), tropospheric heating (Butler et al 2010)

Chen et al (2007)

1) As the surface drag is reduced, the zonal wind acceleration is barotropic and proportional to the surface wind in the extratropics. Meanwhile, the baroclinic eddies are weakened by the increased barotropic meridional shear, but neither the weakening eddies nor the increased meridional shears are directly implicated in the poleward shift.

2) The increase in the strength of the westerlies in the extratropics leads to faster eddy phase speeds, while the subtropical zonal winds barely change. Hence, the critical latitude for these eddies is displaced poleward.

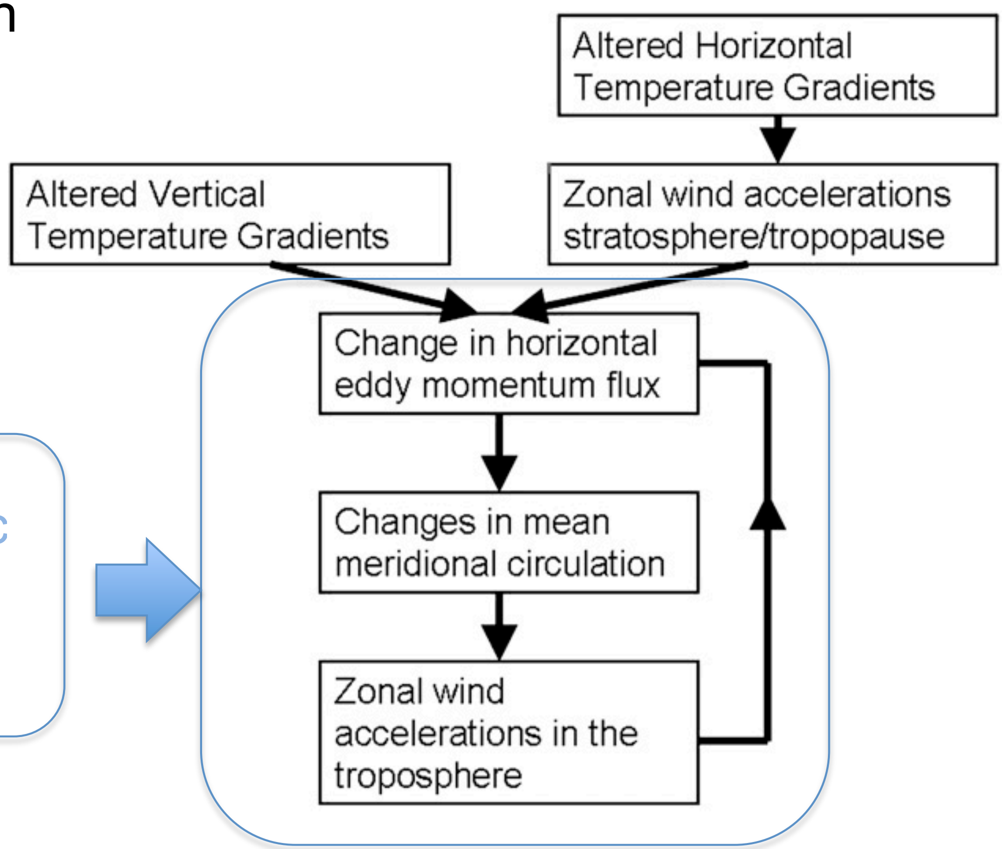
3) The dynamics of the wave breaking in the upper troposphere, in the presence of this poleward shift in critical latitude, shifts the eddy momentum fluxes poleward, driving a poleward shift in the surface zonal winds and the eddy-driven jet. This is particularly supported by the shallow water model results.

4) Eddy heat fluxes, and the associated upward Eliassen– Palm (EP) fluxes tend to follow this upper-level eddy activity. This shift in the baroclinic eddy production provides some positive feedback on the upper-level shift.

Can we make *predictions* about the response of the tropospheric circulation?

Diagram from Simpson et al (2009)

Can this tropospheric box really be ‘decomposed’?

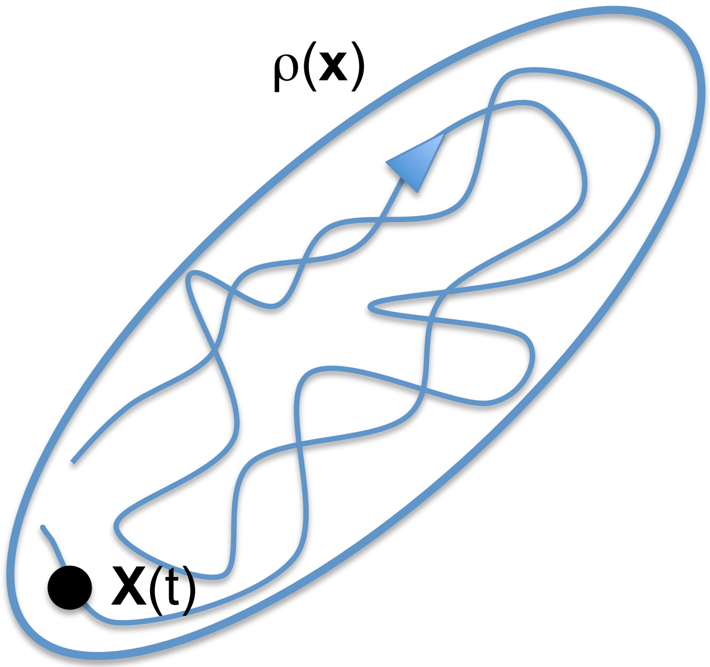


Questions

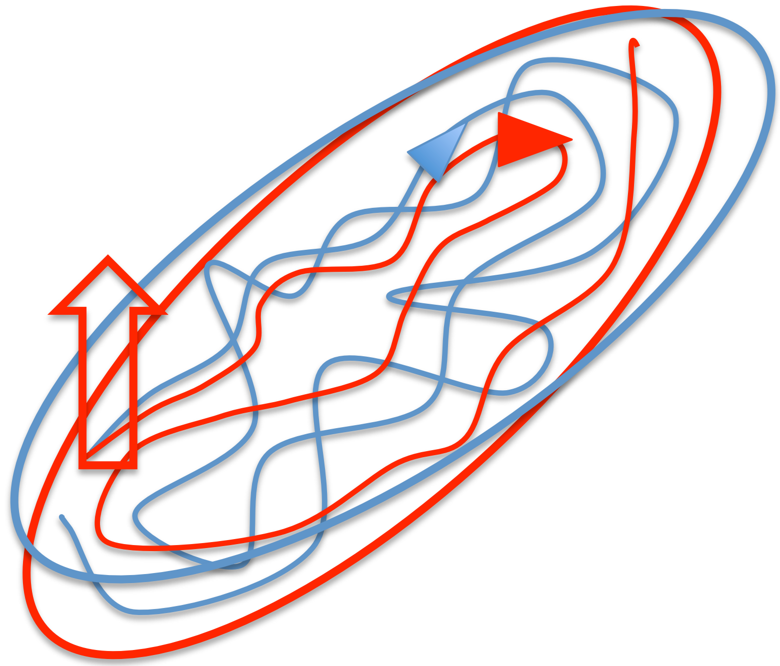
- What is relation between spatial pattern of forcing and the spatial pattern of response? ('preferred response', 'most effective forcing')
- Will different models overpredict or underpredict response relative to real atmosphere?

Seek a 'unified' approach to quantitative prediction of tropospheric response, rather than post-hoc explanation of each special case

Calculation of change in statistical measure of chaotic/ random system due to applied perturbation



UNDISTURBED SYSTEM



DISTURBED SYSTEM

Evolution equation $$\frac{d\mathbf{X}}{dt} = \mathbf{U}(\mathbf{X}, t)$$

$\mathbf{U}(\mathbf{X}, t)$ is usually nonlinear and could contain explicit randomness

Equilibrium statistical properties described by probability density function $\rho(\mathbf{x})$

Perturb $$\frac{d\mathbf{X}}{dt} = \mathbf{U}(\mathbf{X}, t) + \Delta\mathbf{F}(\mathbf{X}, t)$$

What happens to $\rho(\mathbf{x})$?

Consider small applied forcing $\Delta \mathbf{F} = \mathbf{f}(\mathbf{x})\delta(t)$

At $t=0$: $\rho(\mathbf{x}) \to \rho_+(\mathbf{x}) \simeq \rho(\mathbf{x}) - \nabla_{\mathbf{x}}.(\mathbf{f}(\mathbf{x})\rho(\mathbf{x}))$

$$\langle \phi(\mathbf{X}(\tau)) \rangle_{\mathbf{f}} = \int d\mathbf{x} \int d\mathbf{y} \phi(\mathbf{y}) \mathcal{P}(\mathbf{X}(\tau) = \mathbf{y} | \mathbf{X}(0) = \mathbf{x}) \rho_+(\mathbf{x})$$

Compare identity

$$\langle \phi(\mathbf{X}(\tau)) \psi(\mathbf{X}(0)) \rangle =$$
$$\int d\mathbf{x} \int d\mathbf{y} \phi(\mathbf{y}) \mathcal{P}(\mathbf{X}(\tau) = \mathbf{y} | \mathbf{X}(0) = \mathbf{x}) \psi(\mathbf{x}) \rho(\mathbf{x})$$

Hence $\langle \phi(\mathbf{X}(\tau)) \rangle_{\mathbf{f}} \simeq \langle \phi(\mathbf{X}(\tau)) \frac{\rho_+(\mathbf{x})}{\rho(\mathbf{x})} |_{\mathbf{x}=\mathbf{X}(0)} \rangle$ and

$$\boxed{\Delta \langle \phi(\mathbf{X}(\tau)) \rangle = -\langle \phi(\mathbf{X}(\tau)) \frac{\nabla_{\mathbf{x}}.(\mathbf{f}(\mathbf{x})\rho(\mathbf{x}))}{\rho(\mathbf{x})} |_{\mathbf{x}=\mathbf{X}(0)} \rangle}$$

Generalise to arbitrary $\Delta\mathbf{F}(\mathbf{x},t)$ by linear superposition

Hence for steady **x**-independent $\Delta\mathbf{F}$

$$\boxed{\Delta\langle\mathbf{X}\rangle = -\int_0^\infty d\tau \langle \mathbf{X}(\tau)\frac{\nabla_{\mathbf{x}}.\rho(\mathbf{x})}{\rho(\mathbf{x})}|_{\mathbf{x}=\mathbf{X}(0)}\rangle.\Delta\mathbf{F} \;= \mathbf{L}\Delta\mathbf{F}}$$

A

If **X** is Gaussian then $\boxed{\Delta\langle\mathbf{X}\rangle = \int_0^\infty d\tau \mathbf{C}(\tau)\mathbf{C}(0)^{-1}.\Delta\mathbf{F}}$

B

where $\mathbf{C}(\tau) = \langle\mathbf{X}(\tau)\mathbf{X}(0)\rangle$

B is the standard Gaussian form of the Fluctuation-Dissipation Theorem

A is a more general (non-Gaussian) form of the Fluctuation-Dissipation Theorem

Fluctuation-Dissipation Theorem predicts linear response of a random system to applied forcing using information about undisturbed system

$$\Delta\langle\mathbf{X}\rangle = -\int_0^\infty d\tau \langle \mathbf{X}(\tau) \frac{\nabla_{\mathbf{x}}\rho(\mathbf{x})}{\rho(\mathbf{x})}|_{\mathbf{x}=\mathbf{X}(0)}\rangle . \Delta\mathbf{F} \;\; = \mathbf{L}\Delta\mathbf{F}$$

A

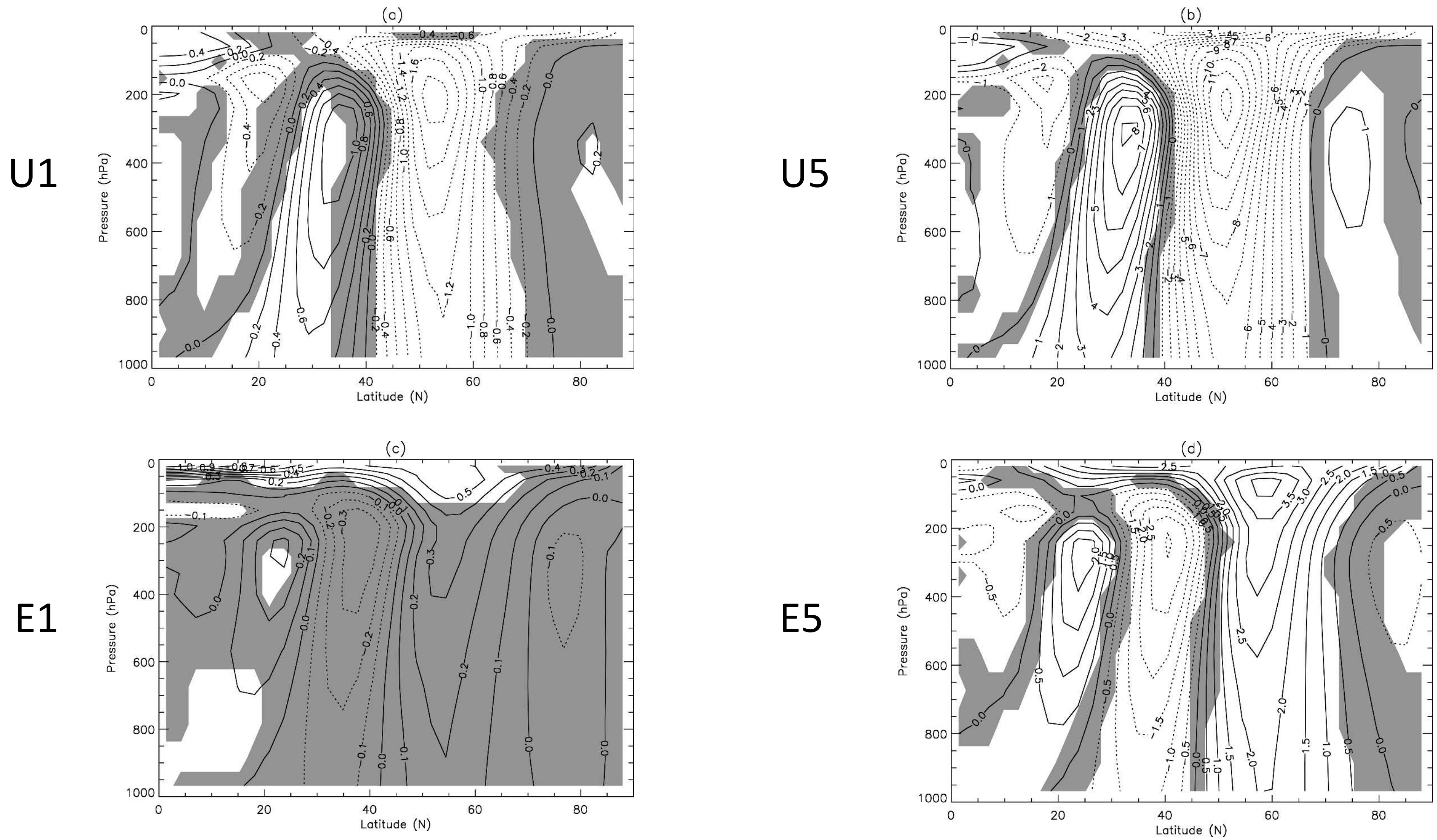
$$\Delta\langle\mathbf{X}\rangle = \int_0^\infty d\tau \mathbf{C}(\tau)\mathbf{C}(0)^{-1}.\Delta\mathbf{F}$$

where $\mathbf{C}(\tau) = \langle\mathbf{X}(\tau)\mathbf{X}(0)\rangle$

B

Usefulness of linear theory?

depends on problem being considered – but recall Haigh et al (2005)

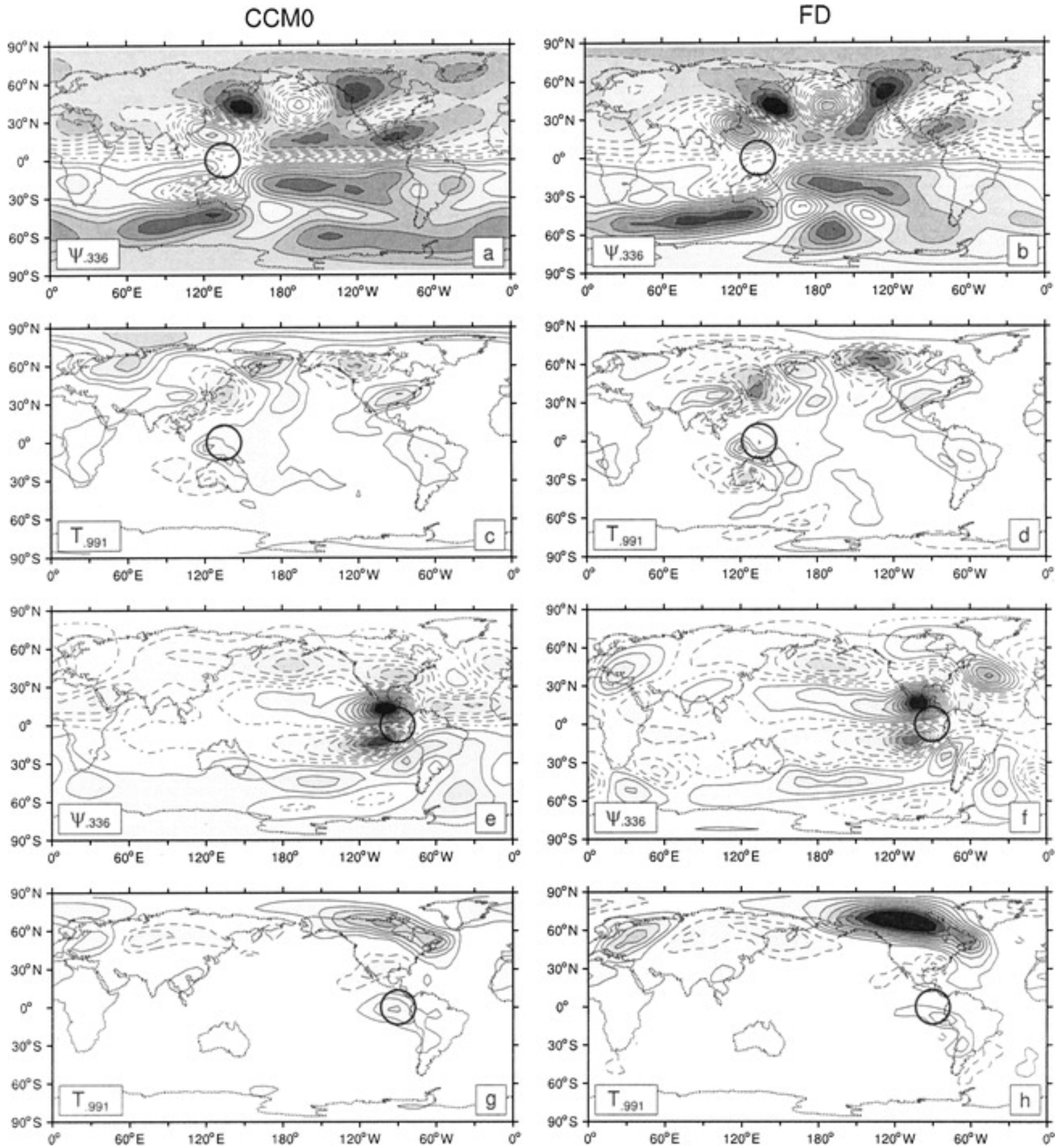


Gritsun and Branstator (2007)

Application of Gaussian FDT to predict response to localised tropical heating in GCM

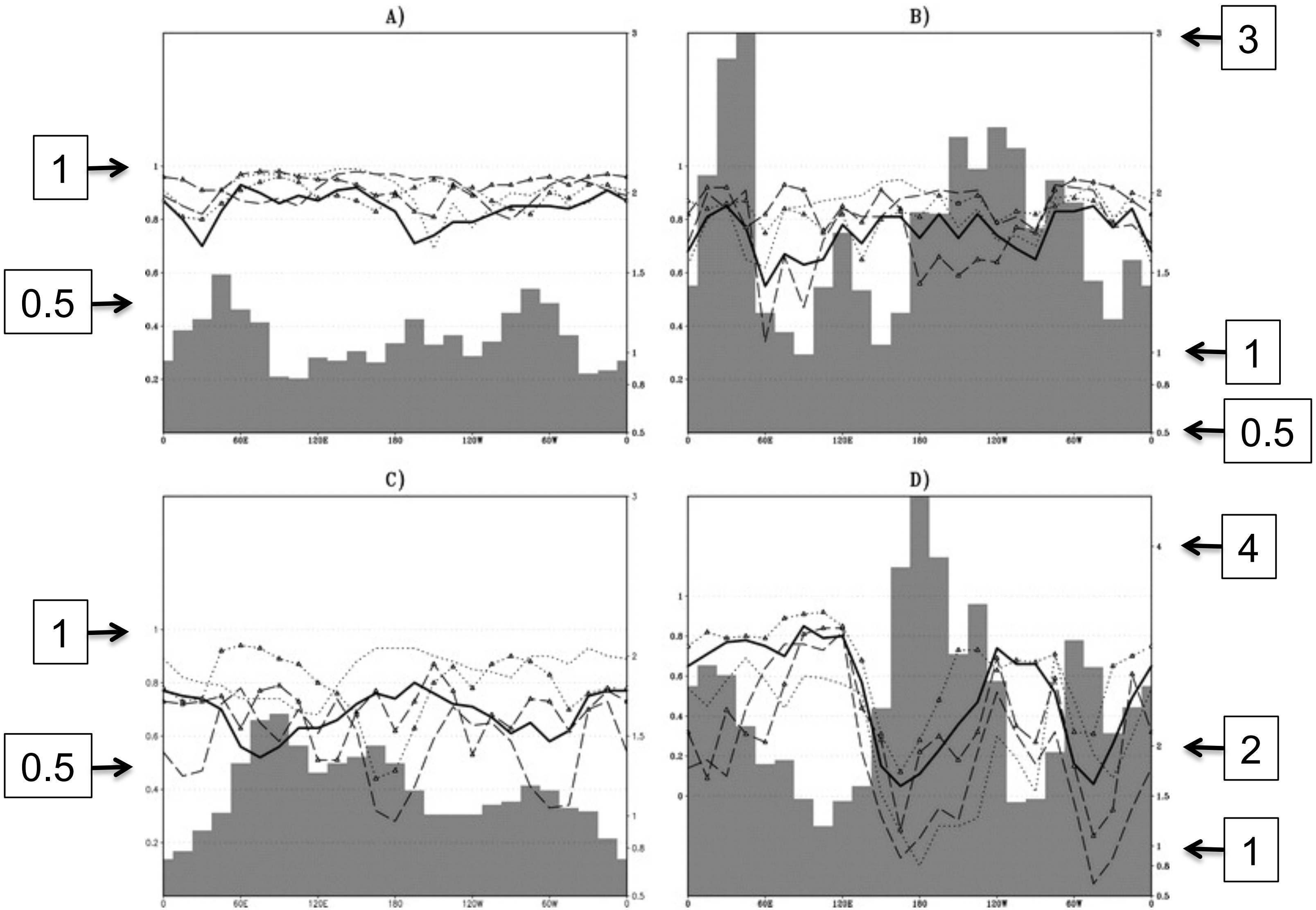
Individual AGCM integrations 40000 days

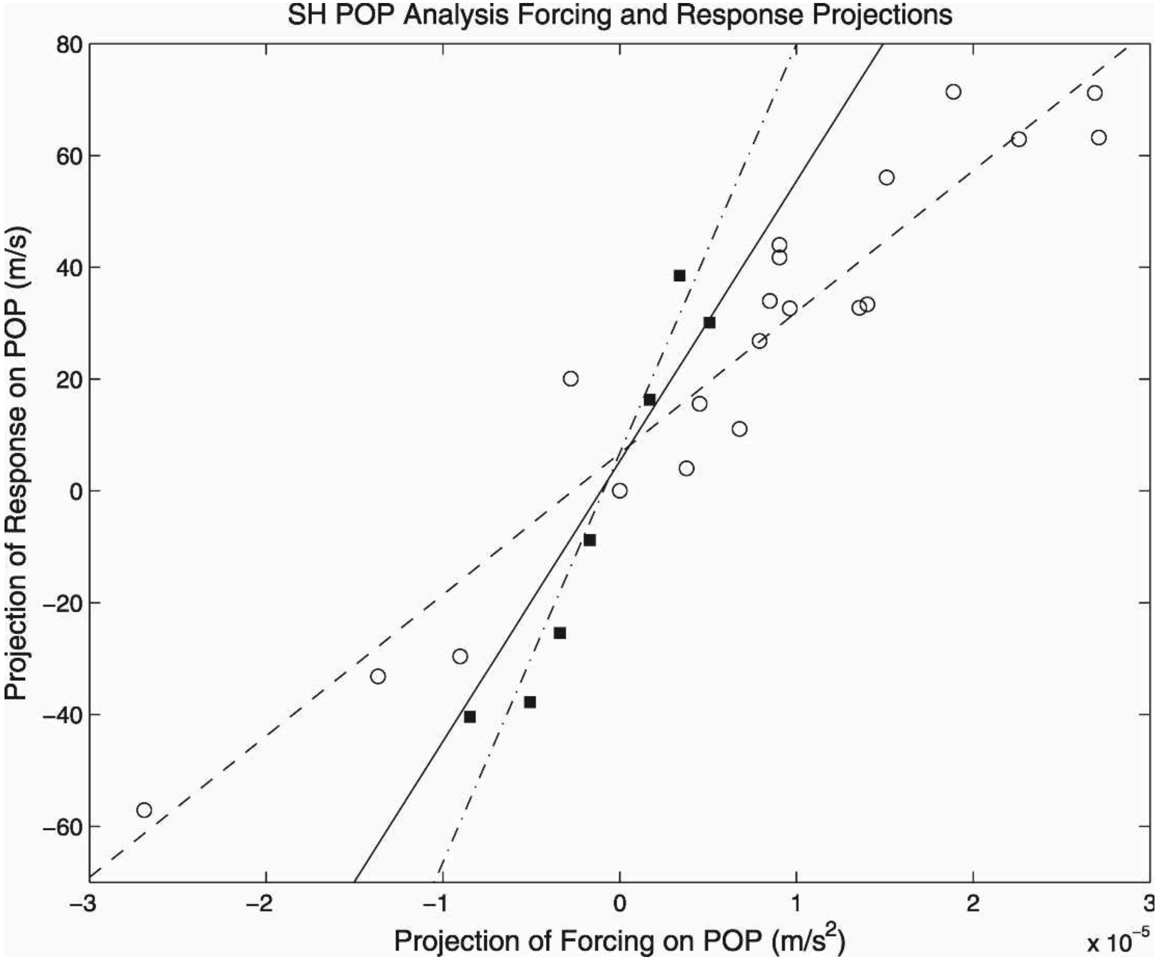
FDT estimate constructed from 4M day integration



Gritsun and Branstator (2007)

Success of FDT measured by pattern correlation and amplitude ratio.





Ring and Plumb (2008): Gaussian FDT makes incorrect prediction for response to zonally symmetric thermal (■) and mechanical (**O**) forcings

Practical issues in applying the FDT

- EOFs (which diagonalise **C**(0)) are a natural choice of variable (but not the only possible choice)

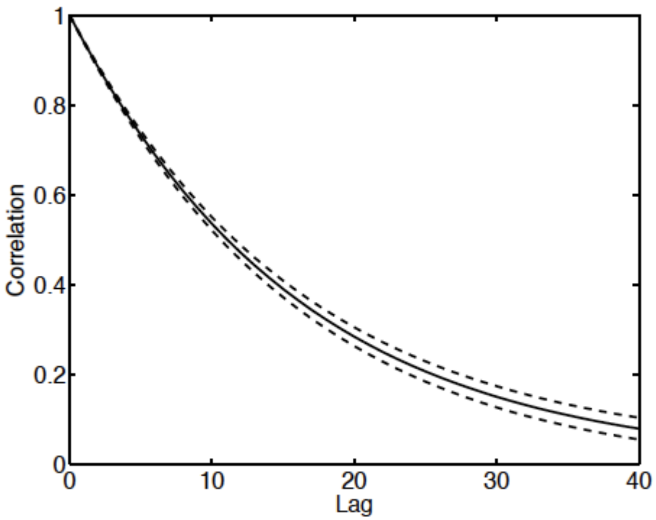
- < **C**(τ) **C**(0)$^{-1}$> must be estimated from available data.
- **C**(0)$^{-1}$ potentially ill-conditioned – number of useful EOFs may be restricted by length of data series

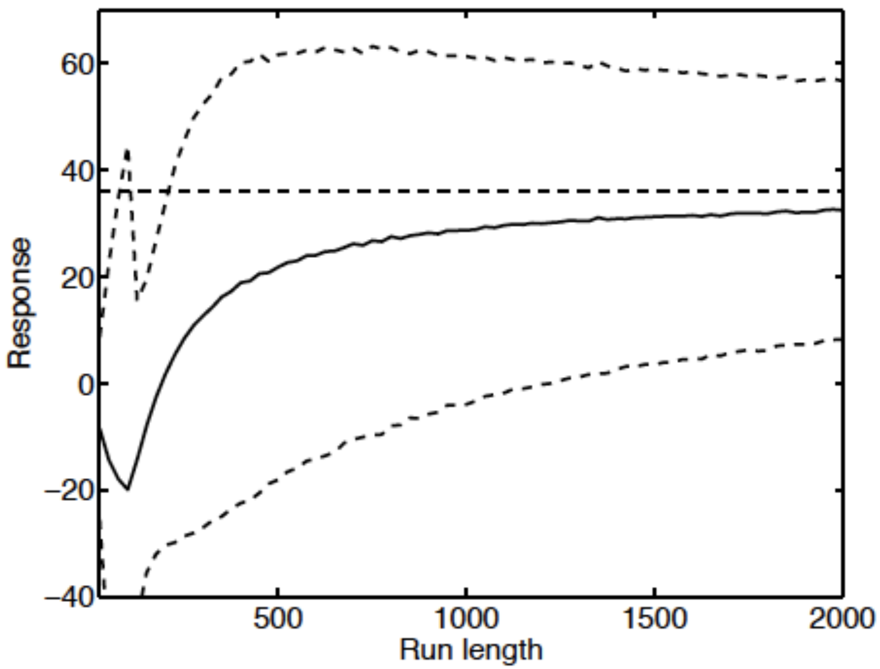
- integration from $\tau = 0$ to $\tau = \infty$ must be approximated by finite sum

Statistical requirements on application of Gaussian FDT

Cooper and H 2012

2-D linear model

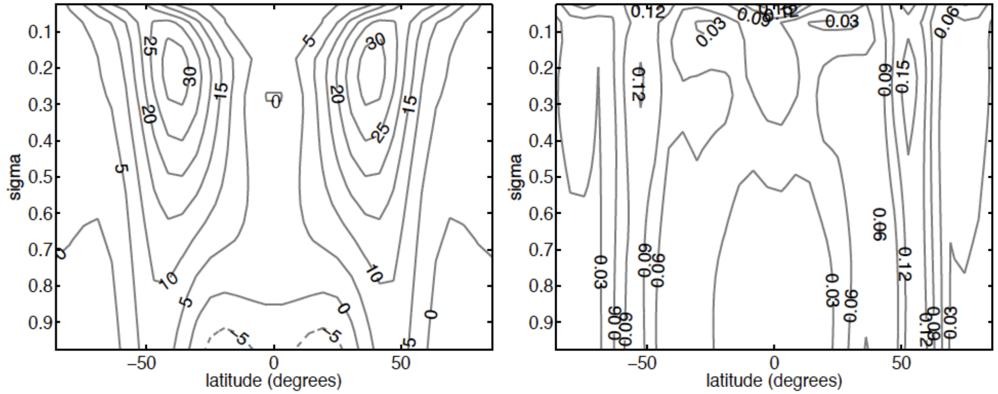




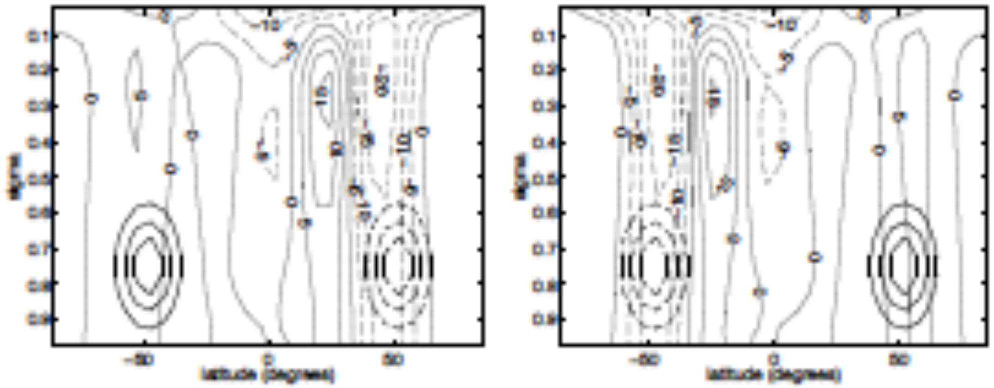
FDT prediction of response

Study based on simple T21L20 general circulation model

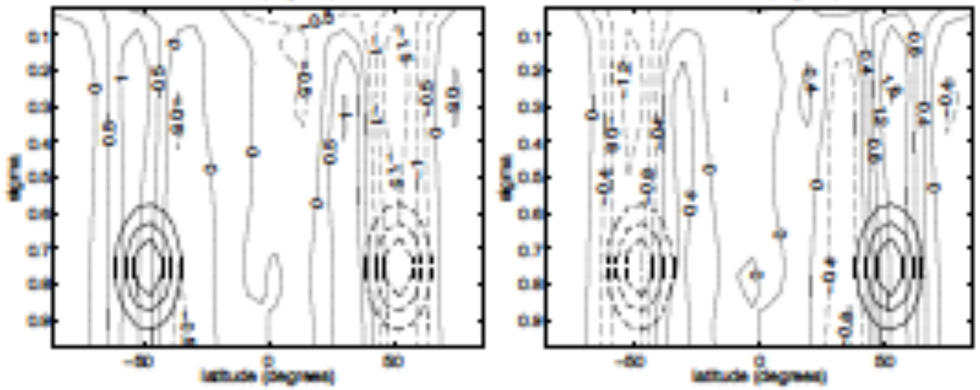
10000 day simulations, mean and variance



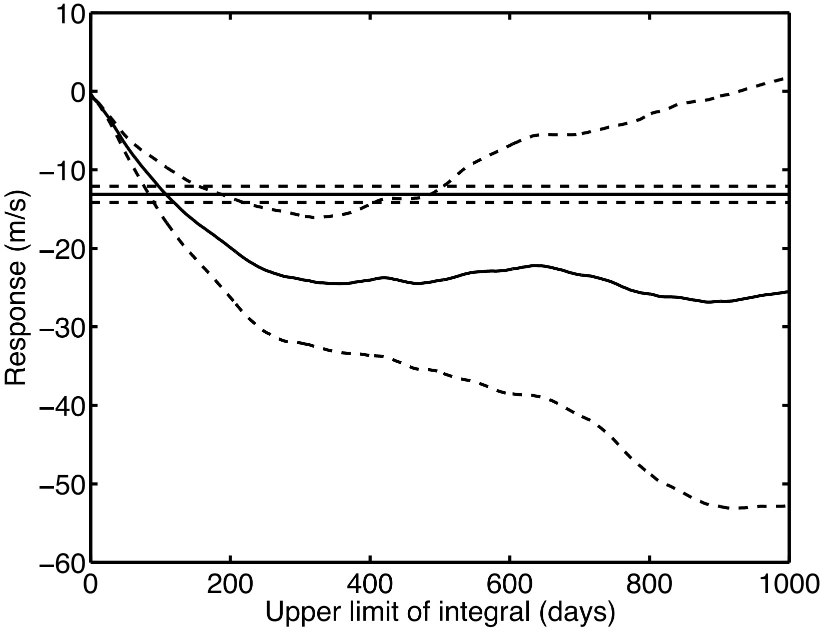
Forced response +/- 1 m/s/day

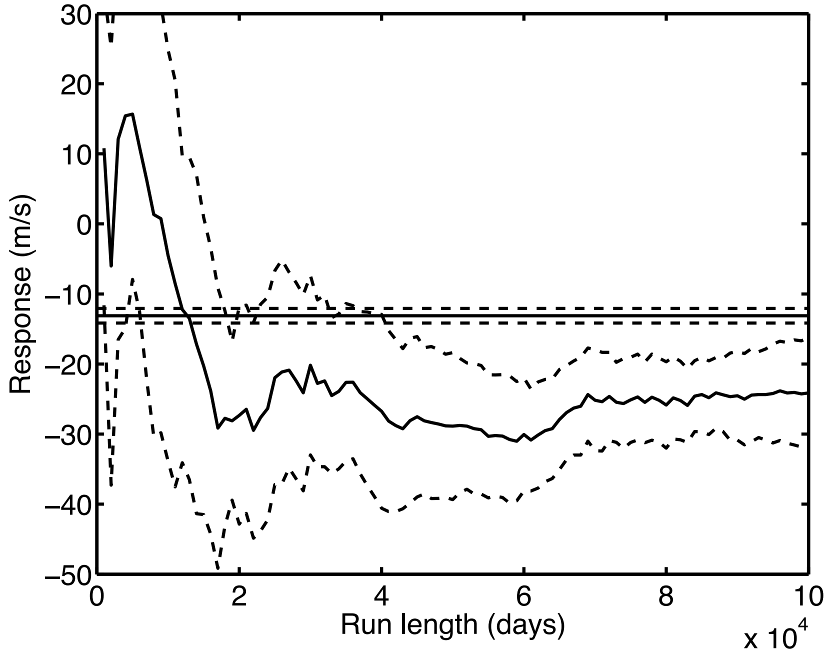


Forced response +/- 0.1 m/s/day



Application of the FDT to predict the response to forcing of a simple T21L20 general circulation model



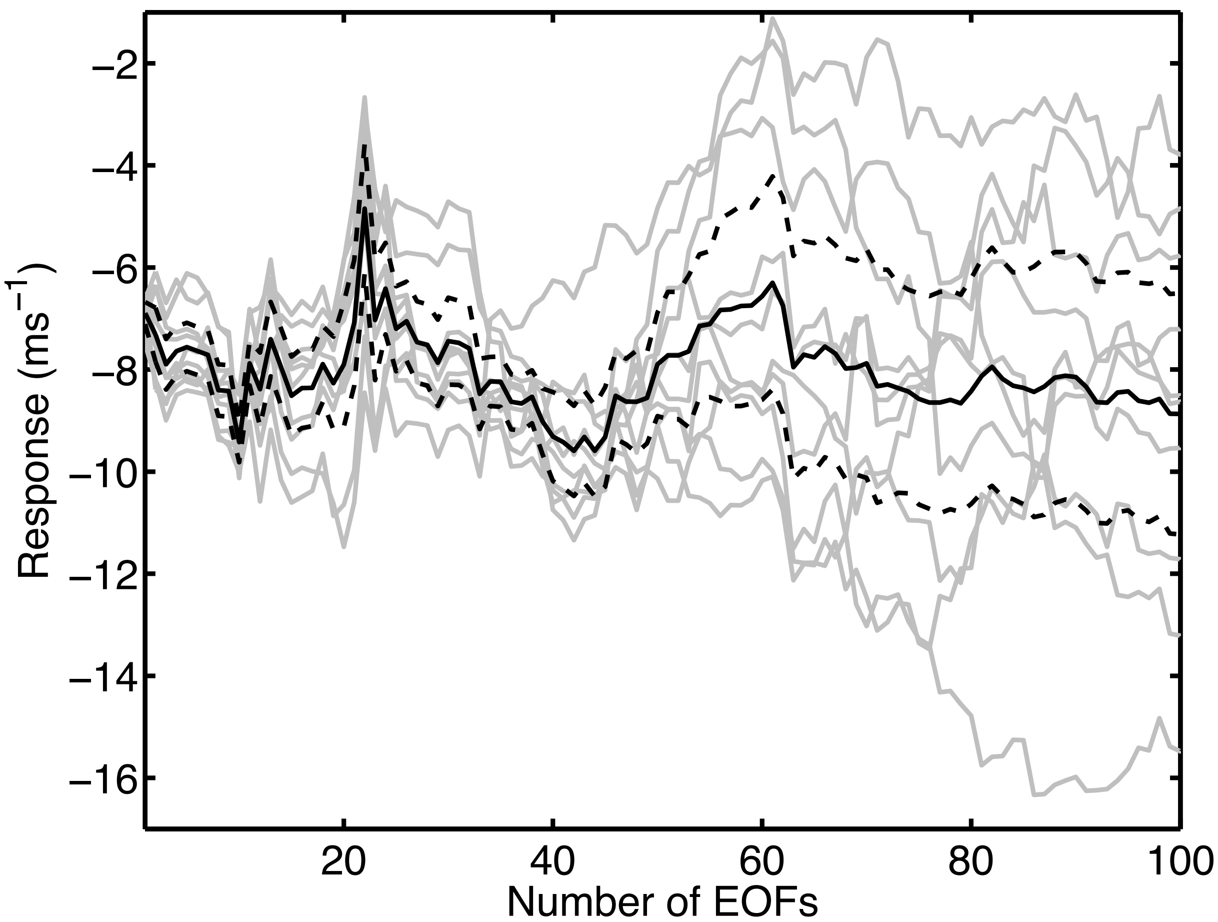


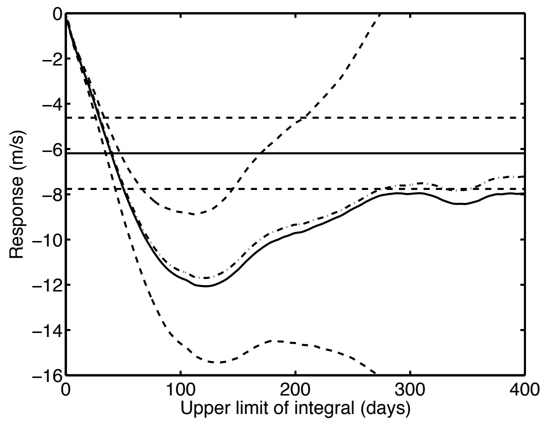
(10 x 10^5 day integrations)

(300 day upper limit to integral)

What is the optimal number of EOFs to include in the calculation?

(T31L20 model)





Specification of forcing

$$\mathbf{L}_{\mathrm{Gaussian}} = \int_0^\infty d\tau \mathbf{C}(\tau)\mathbf{C}(0)^{-1}$$

Where did the equations go?

Implications of truncation

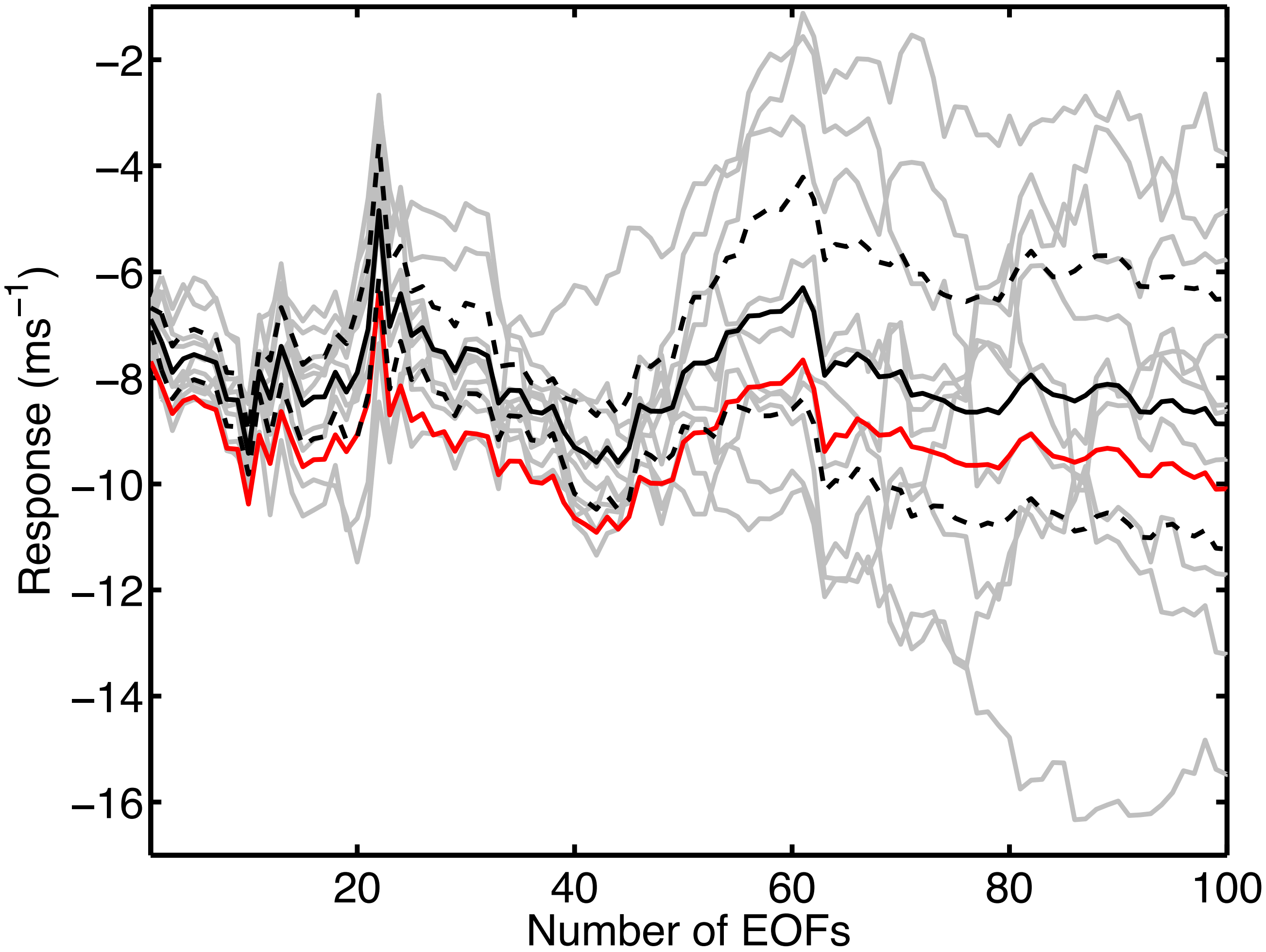
e.g. Ring and Plumb (2007)

$$\frac{\partial u}{\partial t} + \cdots = \mathcal{F} \qquad \frac{\partial T}{\partial t} + \cdots = 0$$

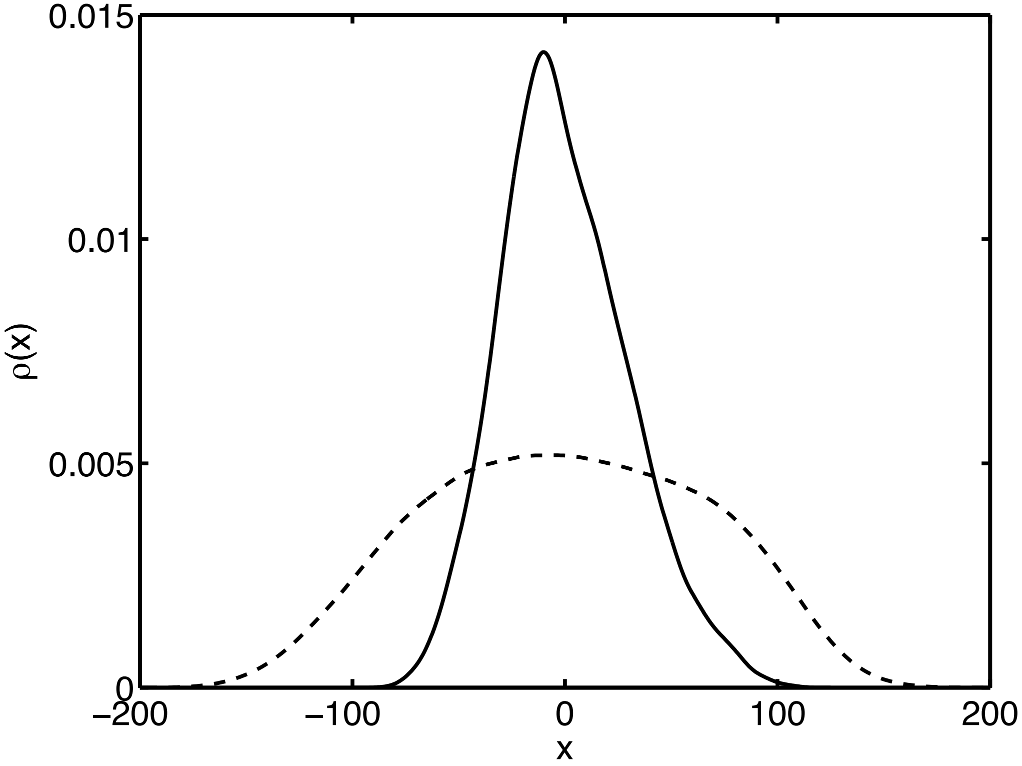
$$\frac{\partial u}{\partial t} + \cdots = \mathcal{L}(u, \mathcal{F})$$

Effect of using climatological u versus u=0 in

$$\mathcal{L}(u, \mathcal{F})$$



Usefulness of Gaussian FDT may be limited by non-Gaussianity?



pdfs of EOF1 in zonal wind for T21L20 and T31L20 simulations

The non-Gaussian case – a 'non-parametric FDT'

Cooper and H 2011

$$\Delta\langle\mathbf{X}\rangle = -\int_0^\infty d\tau \langle \mathbf{X}(\tau)\frac{\nabla_{\mathbf{x}}\rho(\mathbf{x})}{\rho(\mathbf{x})}|_{\mathbf{x}=\mathbf{X}(0)}\rangle . \Delta\mathbf{F}$$

Estimate using kernel density estimator
method of non-parametric statistics

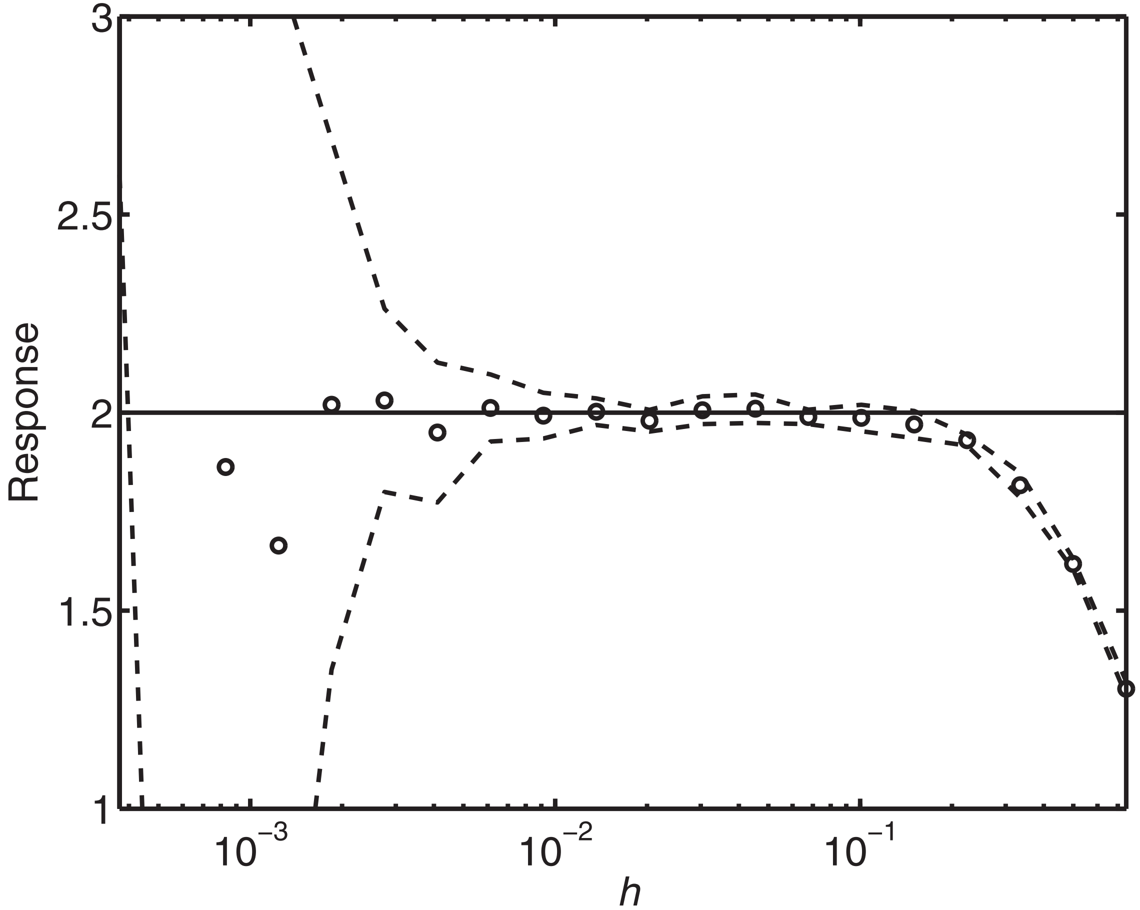
$$\hat{\rho}(\mathbf{x};h,N) = \frac{1}{Nh^d}\sum_{i=1}^{N} K(\frac{\mathbf{x}-\mathbf{X}_i}{h})$$

$$\nabla_{\mathbf{x}}\hat{\rho}(\mathbf{x};h,N) = \ldots$$

Simplest choice for K(.) is
isotropic Gaussian

Bias and *Uncertainty* depend on *h* and *N*.

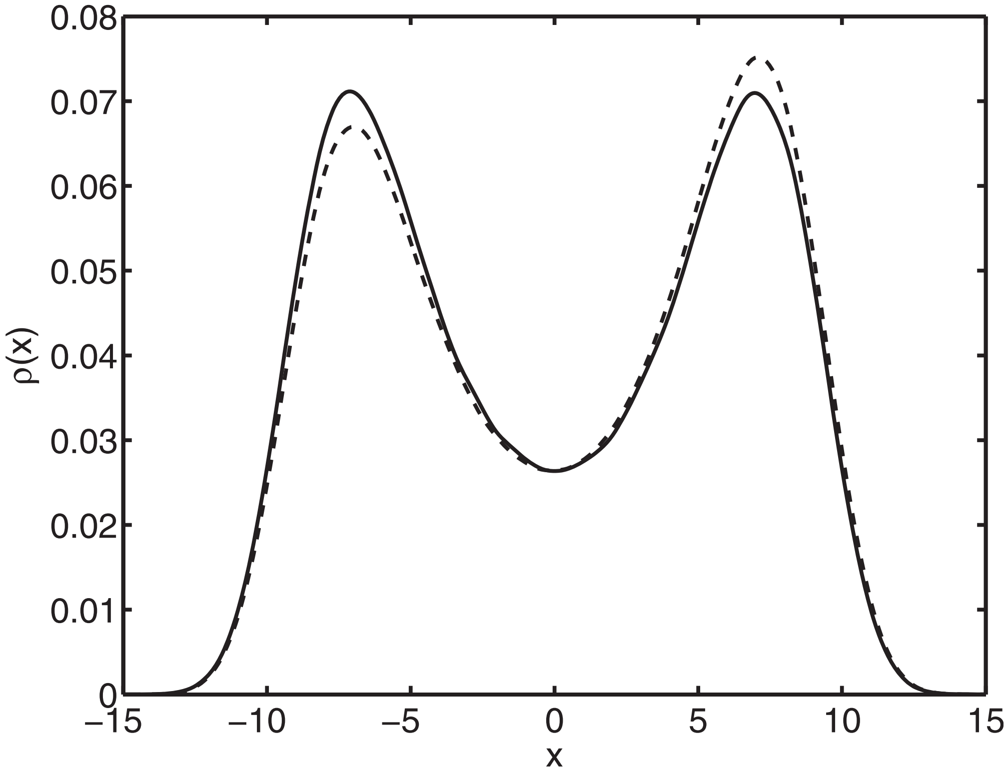
Test case $\frac{d\mathbf{X}}{dt} = \mathbf{B}\mathbf{X} + \xi$

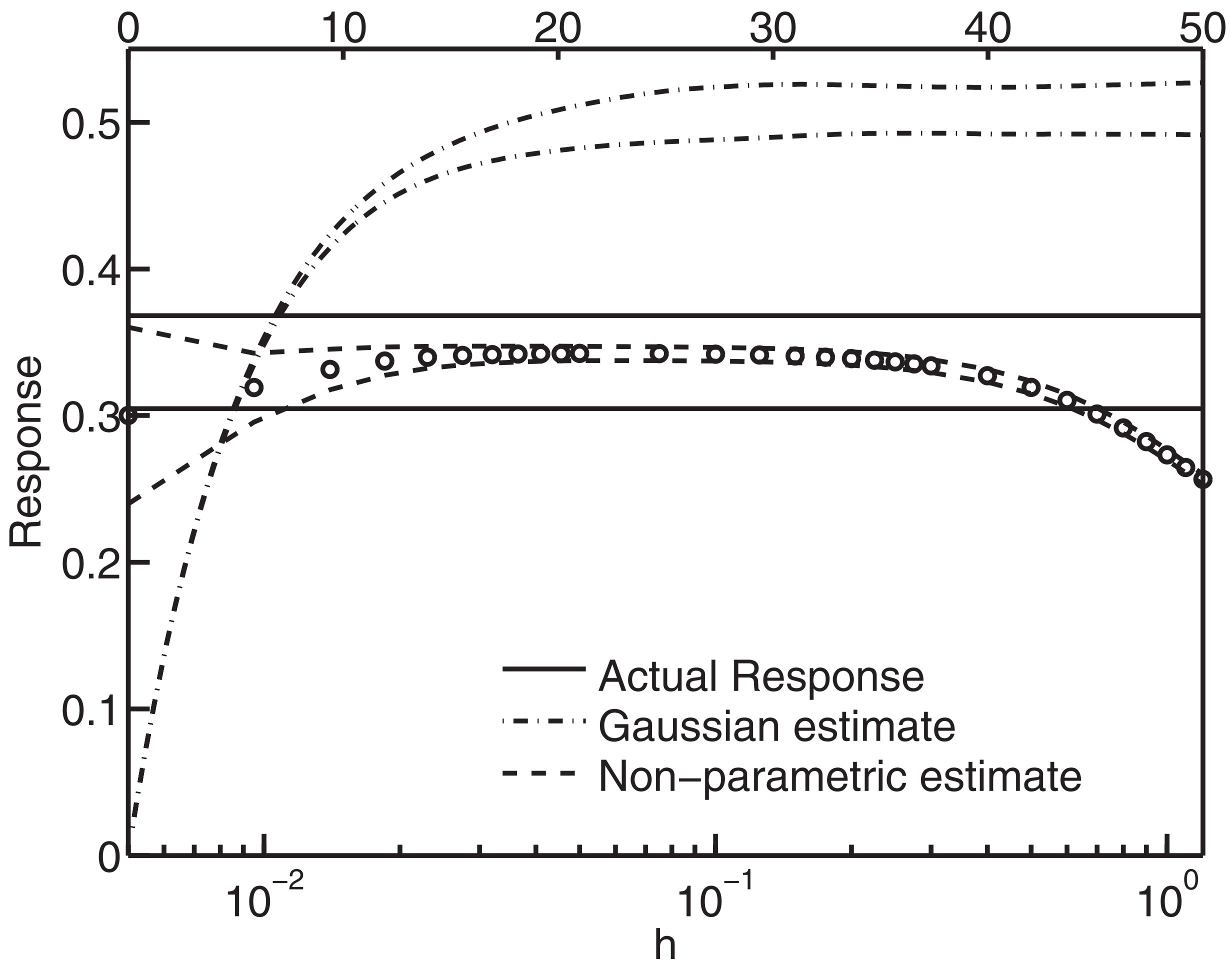


strongly non-Gaussian test case

$$\frac{dX}{dt} = b_1 X - b_2 X^3 + \xi$$

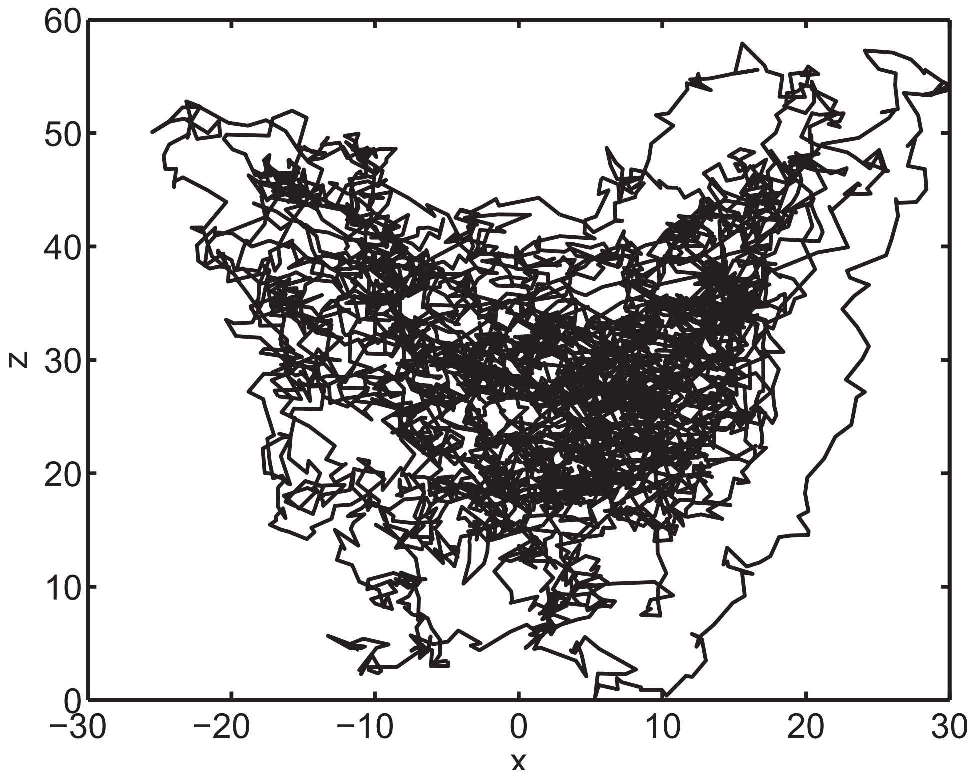
Perturbed and unperturbed pdfs





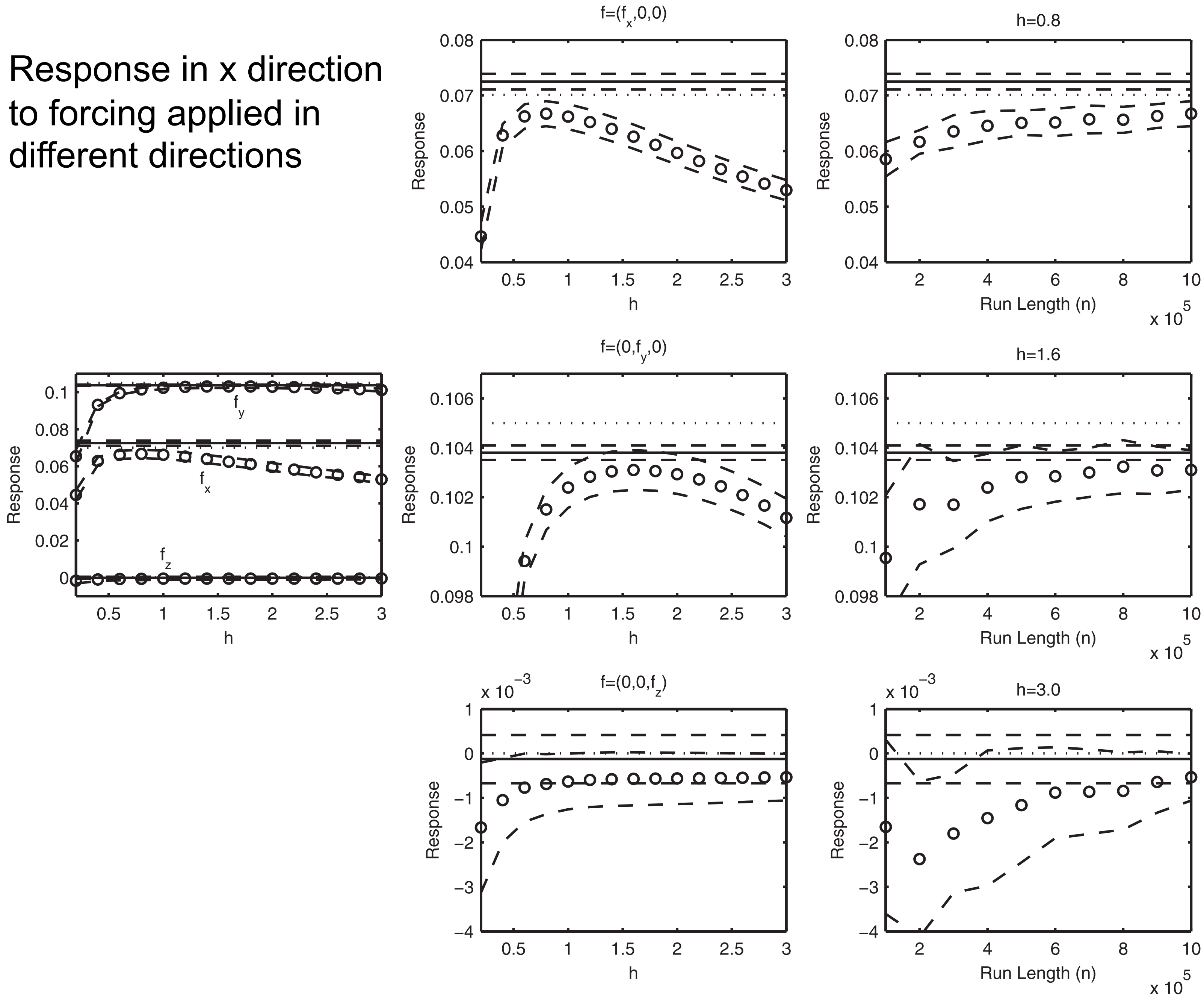
0
10
20
30
40
50
0.5
0.4
0.3
0.2
0.1
0
Response
Actual Response
Gaussian estimate
Non−parametric estimate
10^{-2}
10^{-1}
10^{0}
h

Application to stochastic Lorenz 1963 model

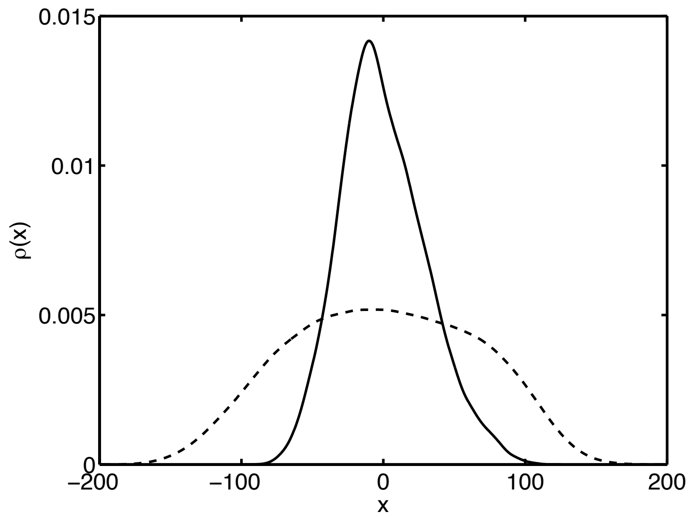


Compare with Thuburn (2005) approach of solving Fokker-Planck equation

Response in x direction to forcing applied in different directions



What is a useful measure of non-Gaussianity?



$$\mathbf{L} - \mathbf{L}_{\mathrm{Gaussian}} = \int_0^\infty d\tau \langle \{ \langle \mathbf{X}(\tau) | \mathbf{X}(0) \rangle \{ -\rho(\mathbf{x})^{-1} \nabla_{\mathbf{x}} \rho(\mathbf{x})|_{\mathbf{x}=\mathbf{X}(0)} - \mathbf{X}(0).\mathbf{C}(0)^{-1} \} \rangle$$

Depends on structure of time correlations as well as form of pdf

Summary

•FDT potentially provides a quantitative description of tropospheric response to forcing (e.g. ozone hole, solar cycle, greenhouse gas increase) given information on statistics of unforced circulation

•Can it do better than simple estimation of inverse timescale of leading EOF?

•Applications? Model assessment/intercomparison

•Statistical nature of FDT requires explicit information on/ estimates for bias and uncertainty

•Non-gaussian extension of FDT potentially extends validity (but there are challenges in implementation)

Future lines of work?

- Clearer practical guide to implementation of FDT (How long a data record is needed for required precision? How many degrees of freedom to include?)

- Non-Gaussian FDT – can we escape the ‘curse of dimensionality’ or avoid it by working in a small number of dimensions?

- Resolution of the ‘thermal vs mechanical forcing’ problem identified by Ring and Plumb (2008)