Elasticity Day 2019
Programme and Abstracts

Wednesday 8th May 2019

University of Exeter

All talks will take place in Streatham Court B. You can view the location of Streatham Court, as well as the facilities available in Lecture Theatre B here. Lunch and coffee will be provided in the foyer outside Streatham Court B.

9:00 - 9:35	**Coffee Break**
9:35 - 9:40	Welcome
9:40 - 10:00	Tunable magnetoelastic phononic crystals, *Emily Glover*
10:00- 10:20	Antiplane shear waves in quasicrystalline metamaterials: negative refraction and self-similarity of the spectrum, *Lorenzo Morini*
10:20 - 10:40	Engineering symmetry-protected topological valley networks in structured elastic plates, *Mehul Makwana*
10:40-11:00	Bridging waves on a membrane: An approach to preserving wave patterns, *Peter Wootton*
11:00 - 11:30	**Coffee Break**
11:30 - 11:50	Local averages of the stress and strain in linear elasticity theory, *Mitchell Berger*
11:50 - 12:10	Graded metasurfaces for wave manipulation on thin elastic plates, *Gregory Chaplain*
12:10 - 12:30	Bespoke nonlinear elasticity, *Isaac Vikram Chenchiah*
12:30 - 12:50	Elastic metamaterials, *Richard Craster*
12:50 - 14:00	**Lunch**
14:00 - 14:20	On the buckling of an embedded spherical shell, *Mike Smith*
14:20 - 14:40	Surface wave fields induced by internal sources, *Anastasiia Chevrychkina*
14:40 - 15:00	A new microstructural strain energy function for the hyperelastic modelling of skin, *James Haughton*
15:00 - 15:20	Modelling large scale metamaterials for elastic waves control, *Bogdan Ungureanu*
15:20 - 15:50	**Coffee Break**
15:50 - 16:10	Para-universal relations for additively split orthotropic constitutive models, *Andrey V. Melnik*
16:10 - 16:30	An efficient semi-analytical scheme for determining the scattering of Lamb waves in a wave-guide with discontinuous depth, *Robert Davey*
16:30 - 16:50	Coupled Scholte modes in soft solid plates, *Beth Staples*
16:50 - 17:00	Closing Remarks

Tunable magnetoelastic phononic crystals

Emily Glover
School of Physics, University of Exeter

The longitudinal elastic modes of a magnetoelastic phononic crystal can tuned by adjusting the lattice structure, and switched on and off by application of a DC magnetic bias field. In this work, the longitudinal modes of a ribbon-shaped magnetoelastic phononic crystal were calculated using a finite element model, and observed in absorption measurements driven by a radio-frequency magnetic field.

Antiplane shear waves in quasicrystalline metamaterials: negative refraction and self-similarity of the spectrum

Lorenzo Morini
School of Engineering, Cardiff University

We investigate the problem of an antiplane wave obliquely incident at the interface between an elastic substrate and a laminate. The considered multilayered media possess a quasicrystalline structure, generated according to the Fibonacci substitution rules [1]. The substrate-laminate system is studied combining the transfer matrix method to the normal mode decomposition technique [2]. The diffraction angles associated with the transmitted modes are estimated by means of the space averaging procedure of the Poynting vector [3]. We show that, with respect to a periodic classical bilayer [4], on the one hand, beyond a certain frequency threshold, high order Fibonacci laminates can provide negative refraction for a wider range of angles of incidence, on the other, they allow negative wave refraction at lower frequencies. Moreover, the performed numerical results illustrate that the Bloch-Floquet spectrum corresponding to this class of laminates has a self-similar character linked to the specialisation of the Kohmoto's invariant, a function of the frequency that was recently studied by the authors for periodic one-dimensional quasicrystalline-generated waveguides [5]. This function is able to explain two types of scaling occurring in dispersion diagrams. The obtained results represent an important advancement towards the realisation of multilayered quasicrystalline metamaterials.

References:

[1] Gei M. Waves in quasiperiodic structures: stop/pass band distribution and prestress effects, Int. J. Solids Struct., 47 (2010) 3067-3075.

[2] Morini L. , Eyzat Y. and Gei M. Negative refraction in quasicrystalline multilayered metamaterials, J. Mech. Phys. Solids, 124 (2019) 282-298.

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[4] Srivastava A. Metamaterial properties of periodic laminates, J. Mech. Phys. Solids, 96 (2016) 252-263.

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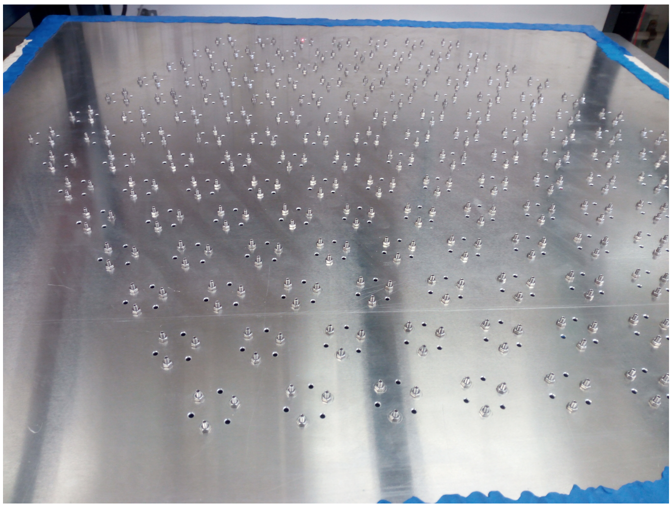
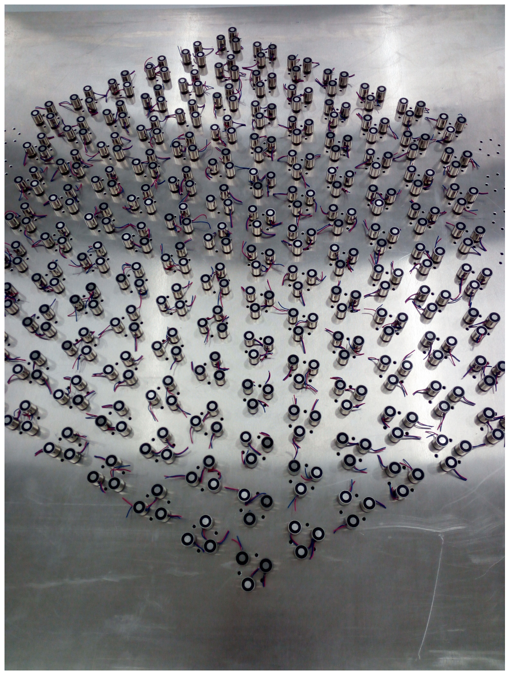
Engineering symmetry-protected topological valley networks in structured elastic plates

Mehul Makwana
Department of Mathematics, Imperial College London

Guiding waves, splitting, and redirecting them between channels, and steering waves around sharp bends in a robust and lossless manner is of interest across many areas of engineering and physics. Recent advances based upon ideas originating from topological insulators , translated to Newtonian wave systems, have inspired great interest. In particular, geometrically engineering topological elastic crystals to direct waves along interfaces in a robust tuneable manner has shown much potential.

Herein, we elucidate the core concepts from topological insulators and apply those to elasticity as well as demonstrate recent advances made in creating large scale topological networks. Within these, the directionality is tunable purely by geometry. The elegance of these designs make them ideal for applications such as beam-splitters, switches and filters. Preliminary experimental results for a structured elastic plate will be shown (see figures below).





Bridging waves on a membrane: An approach to preserving wave patterns.

Peter Wootton, Julius Kaplunov
School of Computing and Mathematics, Keele University

We introduce a novel metamaterial intended to 'bridge' a gap between two 2D membranes using periodic arrays of 1D strings, with the aim of identically reproducing an incident wave form on the other side of the void. This involves both homogenisation and an exact Fourier series to treat the connection between the two materials. While this scheme is tunable, the arrays are not necessarily broadband or successfully bridge waves at multiple incident angles. However the simplicity of the construction gives scope for further development and extension to other media.

Local averages of the stress and strain in linear elasticity theory

Mitchell Berger
Department of Mathematics, University of Exeter

The elastic properties of structured materials are often averaged over sub-volumes of various scales inside the material. Simple volume averaging of the stress and strain may not preserve the elastic energy under arbitrary boundary conditions. We introduce an averaging process which preserves the energy for all boundary conditions. This averaging process emphasizes the parts of the material which are most dynamically important during a deformation. Here the effective strain is weighted by the local stress, and can be interpreted as an average strain over all paths taken by loads and forces through the volume. This alternative averaging process may be especially appropriate for materials with voids such as foams and granular matter, as the averaging only involves the material itself. The effective strain matches the expected engineering strain for a regular honeycomb.

Graded metasurfaces for wave manipulation on thin elastic plates

Gregory Chaplain
Department of Mathematics, Imperial College London

Structured thin elastic plates are capable of supporting array guided modes, which can be manipulated to produce a myriad of interesting effects for controlling wave propagation. In order to geometrically design such systems we introduce a spectral method for efficiently calculating dispersion relations of structures which support waves characterised by periodic behaviour in one direction and exponential decay in the orthogonal direction. We apply our methodology for the design of graded structures capable of displaying rainbow trapping and mode conversion, and present a new hybrid phenomena with applications in flat lensing through emulating negative refraction on a line.

Bespoke nonlinear elasticity

Isaac Vikram Chenchiah
School of Mathematics, University of Bristol

We present a design for an effectively one-dimensional architectured material, comprised of concentric cylindrical lattices, which can be tailored to have any prescribed continuous energy density to desired accuracy. Thus we are able to design effectively one-dimensional structures with arbitrary force-elongation relationships.

Elastic metamaterials

Richard Craster
Department of Mathematics, Imperial College London

There is much activity at present in the field of metamaterials, having its roots in optics and electromagnetic waves. This talk will briefly outline different strategies to control elastic surface waves and vibration using metamaterial concepts.

On the buckling of an embedded spherical shell

Mike Smith
School of Mathematics, University of Manchester

Syntactic foams are a novel class of composite material comprising an elastomeric matrix with micro-balloons as a filler. Recently, stiff polymeric micro-balloons have emerged as a means to create syntactic foams that possess versatile acoustic, elastic, viscoelastic and thermal properties, for example. One particularly useful property is their strong elastic recoverability under extremely large loadings. In order to understand the macroscale mechanical properties and failure mechanisms of such foams, we investigate the buckling of a constituent sphere when embedded in a rubbery matrix material. Under uniaxial deformation, it is well-known that an isolated elastic spherical shell will undergo snap-through buckling at the poles. Counter-intuitively, we show that an embedded spherical shell under uniaxial deformation undergoes buckling along the equator, using small-strain moderate-rotation (SSMR) shell theory. Our findings have important implications for the microstructural modelling of all-polymeric syntactic foams. This talk is based on joint work with Maria Thorpe, Gareth Wyn Jones, Dave Abrahams, and Will Parnell.

Surface wave fields induced by internal sources

Anastasiia Chevrychkina[1,2], Julius Kaplunov[3], Danila Prikazchikov[3]

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[2] *Institute for Problems in Mechanical Engineering of the Russian Academy of Sciences, St. Petersburg, Russia*
[3] *School of Computing and Mathematics, Keele University*

An explicit hyperbolic – elliptic formulation for the Rayleigh wave has been developing over the last decade, see e.g. [1] and references therein. The formulation has been implemented within various setups, e.g., moving loads, layered structures, pre-stressed and anisotropic media, see also [2 – 4], restricted however only to dynamic loading in the form of prescribed surface stresses. In this presentation, we extend the existing framework to calculation of surface wave fields due to internal sources. The superposition principle is adapted by incorporating the classical solutions for sources embedded into an infinite elastic space. As an example, the effect of point time-harmonic linear and spherical sources is studied.

References

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[4] Nobili, A. and Prikazchikov, D.A., 2018. Explicit formulation for the Rayleigh wave field induced by surface stresses in an orthorhombic half-plane. European Journal of Mechanics-A/Solids, 70, pp.86-94.

A new microstructural strain energy function for the hyperelastic modelling of skin

James Haughton
School of Mathematics, University of Manchester

In this talk, we construct a microstructural strain energy function (SEF) to model the anisotropic and nonlinear stress-strain behaviour exhibited by skin. The SEF is derived by taking account of the microscale deformation of the collagen fibres that are the primary structural component of the skin. We fit the new SEF to uniaxial tensile tests on four different skin samples and compare its quality of fit to that of the HGO SEF, which is widely used in soft tissue modelling. We then incorporate fibre dispersion into the model in order to fit to tensile tests performed in two, perpendicular directions simultaneously. Finally, we account for parameter uncertainty in the new SEF by using Approximate Bayesian Computation, obtaining posterior distributions for model parameters that are consistent with the mechanical data.

Modelling large scale metamaterials for elastic waves control

Bogdan Ungureanu[1], Richard Craster[1], Younes Achaoui[2], Stéphane Brûlé[2], Stefan Enoch[2] and Sébastien Guenneau[3]

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[2]Aix-Marseille Univ, CNRS, Centrale Marseille, Institut Fresnel, Marseille, France
[3]Department of Mathematical Sciences, Liverpool University

We've studied and proposed to apply the large scale metamaterials properties, which are based on the negative values of the Bulk modules K and G; the Young modulus, of longitudinal elasticity, as well as on the negative mass density. These negative values are obtained with the help of the local resonance of the elementary cells that lead to very dispersive properties of the metamaterials.

Other ways of getting such exotic phenomena are by means of exploiting the auxetic metamaterials which have a negative Poisson coefficient, or by the new concept named Metacity, based on the quasiconformal transformations in the elasticity equation.

Para-universal relations for additively split orthotropic constitutive models

Andrey V. Melnik, Xiaoyu Luo, Ray W. Ogden
School of Mathematics and Statistics, University of Glasgow

Many soft biological tissues are modelled as composites consisting of isotropic matrix reinforced by anisotropic fibres, e.g., collagen. It is often assumed in homogenised models of such composites, that the total mechanical response is given by the sum of the contributions of the constituents. The general consequences of this additive split assumption are not studied, and the assumption is maintained, as long as the model predictions reflect experimentally observed behaviour. We find that the additive split leads to a relation for the predicted Cauchy stress, which holds universally in a certain class of anisotropic materials [1]. In the theory of nonlinear elasticity, a *universal relation* is defined is an algebraic relation between stress and deformation that holds for all materials within a certain class, irrespective of the exact form of the material response function and parameter values. The well-known Rivlin's relation for simple shear in isotropic materials $\sigma_{11} - \sigma_{22} = \gamma\sigma_{12}$ [2, 3] and the universal relations established in anisotropic materials [4, 5] apply to stress components produced by *one and the same* deformation. We propose a family of relations that connect stress components under *different* deformations, which we call para-universal relations to highlight this difference. The proposed *para-universal relations* hold for any orthotropic material whose response function is additively decomposed into terms, each of which possesses a symmetry with respect to one of the axes of orthotropy. Like classical universal relations, the proposed para-universal relations hold for all admissible deformations, are linked to material symmetry, and apply to a wide class of materials. These relations can be used to test for the suitability of the additively split strain energy functions. Such a test can be performed on collected experimental data prior to choosing an exact form of the response function and fitting its parameters. We illustrate that using published experimental data for human myocardium and also synthetic data.

References

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An efficient semi-analytical scheme for determining the scattering of Lamb waves in a wave-guide with discontinuous depth

Robert Davey
School of Mathematics, University of Manchester

The reflection and transmission of Lamb waves in an elastodynamic plate with a step change in depth is studied. It is known that extant modal expansion methods for finding the reflected field cannot reconstruct the field near the corners, due to the presence of singularities. We will derive the form of the singularities present near an elastodynamic corner asymptotically. These asymptotics also find the irregular terms, which are terms that are bounded but have expansions in non-integer powers in distance from the corner. These irregular terms are also poorly represented when using Lamb wave expansion methods. To solve the problem of poor representation near the corners we have developed a new method for representing the form of the corner behaviour in its own modes. These new modes are consistent with the Lamb waves and not over complete. Results which show how this new method can accurately represent the near and far field will be presented.

Coupled Scholte modes in soft solid plates

Beth Staples
School of Physics, University of Exeter

Scholte waves are evanescent acoustic waves that propagate along the interface between a liquid and an elastic solid. In a thin plate, interface waves can couple to form a symmetric and antisymmetric pair. Most of the previous studies on coupled Scholte modes deal with plates, such as metals, where only a single antisymmetric mode exists. This study looks at the behaviour of this mode in soft solids, where the transverse wave speed is lower than the speed of sound in the surrounding liquid. This condition allows for a second, symmetric coupled Scholte mode, the existence of which is experimentally verified by using ultrasound pulses to excite both coupled modes in acrylic plates submerged in water.