

# **Probability and the Law**

**Exeter University**

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# Overview

- 1. The cases**
- 2. Probability fallacies and the law**
- 3. The scaling problem and other challenges of Bayes**
- 4. Applying Bayesian networks to the cases**
- 5. Conclusions and way forward**

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# THE CASES

# R v Sally Clark 1999-2003



**Convicted and ultimately cleared of murdering her 2 children**

# R vs Levi Bellfield, Sept 07 – Feb 08



**Amelie Delagrang    Marsha McDonnell**

# R v Gary Dobson 2011



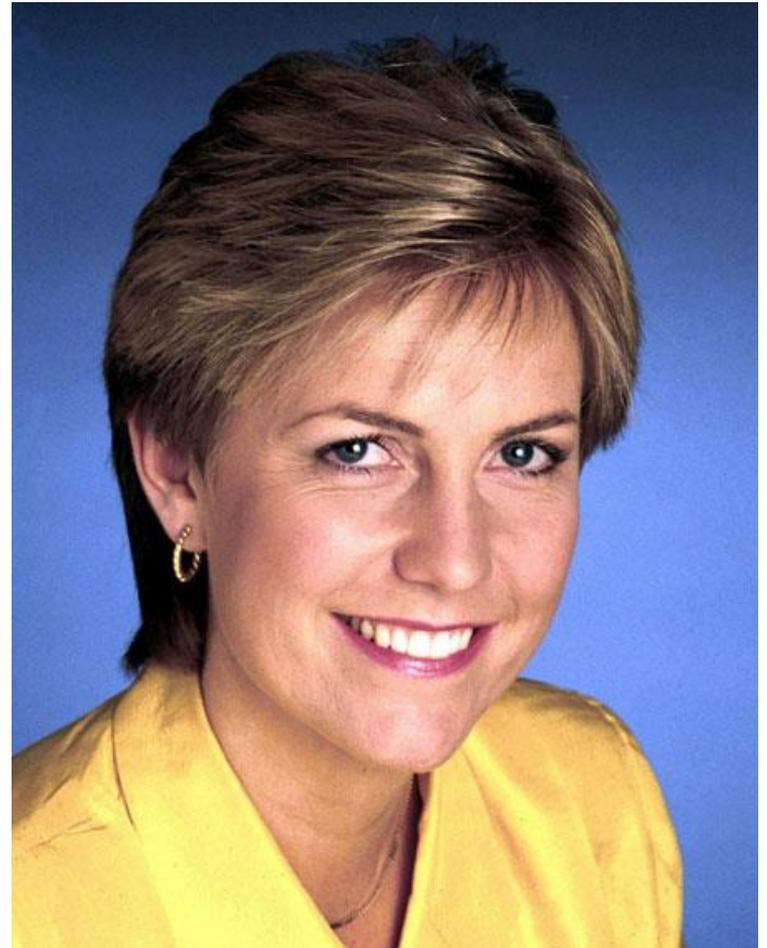
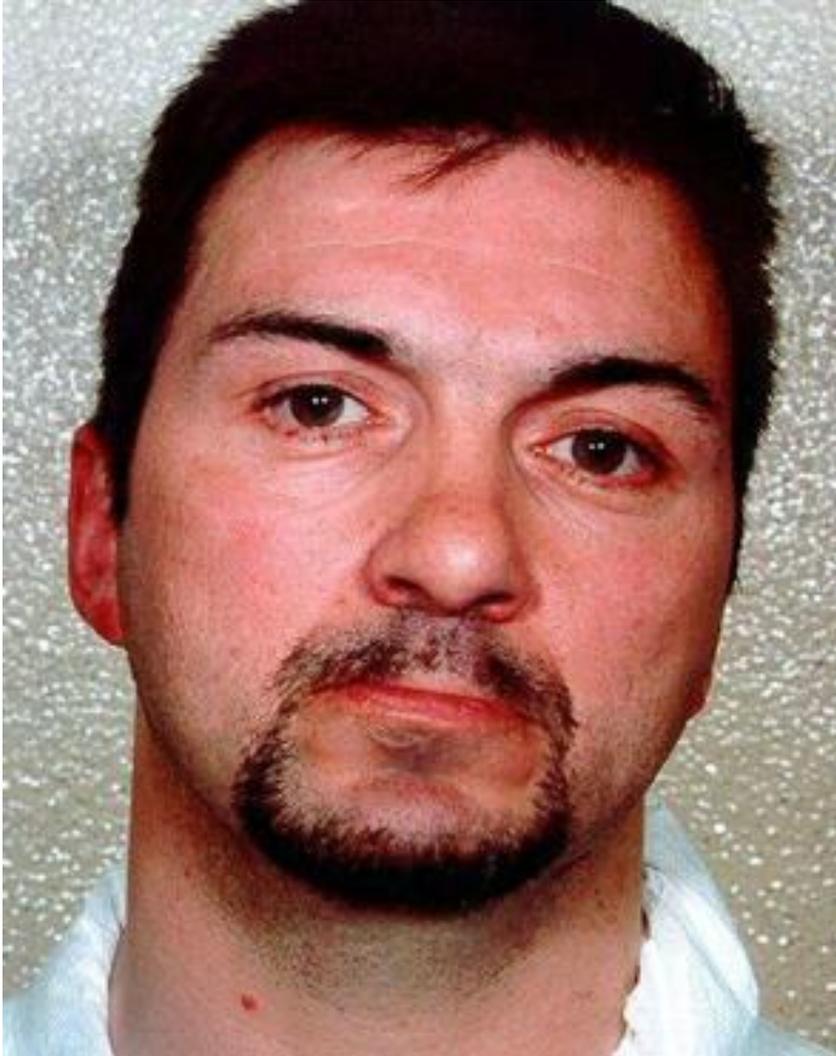
**Stephen Lawrence**

# R v Mark Dixie, 2007-2008



**Sally Anne-Bowman**

# R v Barry George, 2001-2007



**Jill Dando**

2

# **PROBABILITY FALLACIES AND THE LAW**

# Questions

- What is 723539016321014567 divided by 9084523963087620508237120424982?
- What is the area of a field whose length is approximately 100 metres and whose width is approximately 50 metres?

# Court of Appeal Rulings

“The task of the jury is to evaluate evidence and reach a conclusion not by means of a formula, mathematical or otherwise, but by the joint application of their individual common sense and knowledge of the world to the evidence before them” (R v Adams, 1995)

“..no attempt can realistically be made in the generality of cases to use a formula to calculate the probabilities. .. it is quite clear that outside the field of DNA (and possibly other areas where there is a firm statistical base) this court has made it clear that Bayes theorem and likelihood ratios should not be used” (R v T, 2010)

# Revising beliefs when you get forensic 'match' evidence

- Fred is one of a number of men who were at the scene of the crime. The (prior) probability he committed the crime is the same probability as the other men.
- We discover the criminal's shoe size was 13 – a size found nationally in about only 1 in a 100 men. Fred is size 13. Clearly our belief in Fred's innocence decreases. But what is the probability now?

# Are these statements correct/ equivalent?

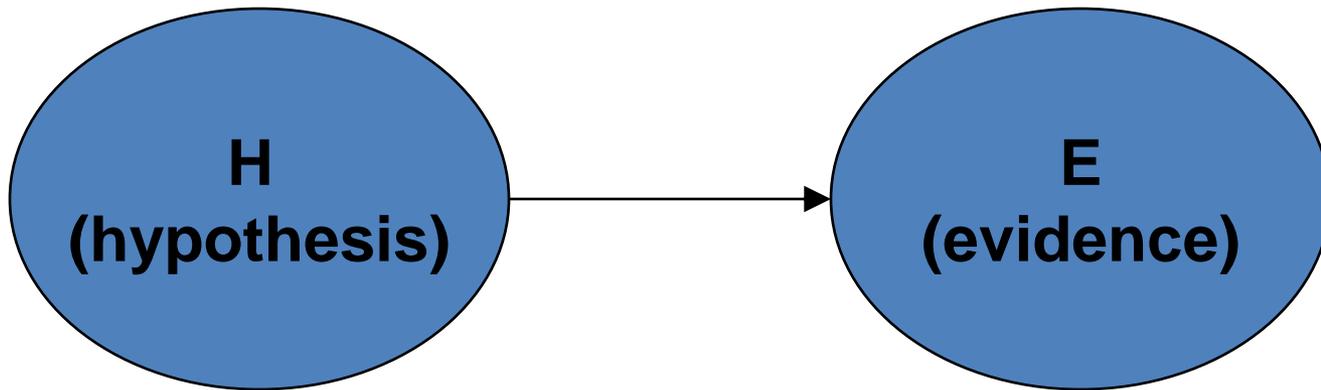
- the probability of finding this evidence (matching shoe size) given the defendant is innocent is 1 in 100
- the probability the defendant is innocent given this evidence is 1 in 100

***The 'prosecution fallacy' is to treat the second statement as equivalent to the first***

# Bayes Theorem

We have a prior  $P(H)$

We now get some evidence  $E$ .



We want to know the posterior  $P(H|E)$

$$P(H|E) = \frac{P(E|H)*P(H)}{P(E)} = \frac{P(E|H)*P(H)}{P(E|H)*P(H) + P(E|\text{not } H)*P(\text{not } H)}$$

$$P(H|E) = \frac{1*1/1001}{1*1/1001 + 1/100*1000/10001} = \frac{0.000999}{0.000999 + 0.00999} \approx 0.091$$

# An intuitive explanation of Bayes for the simple case

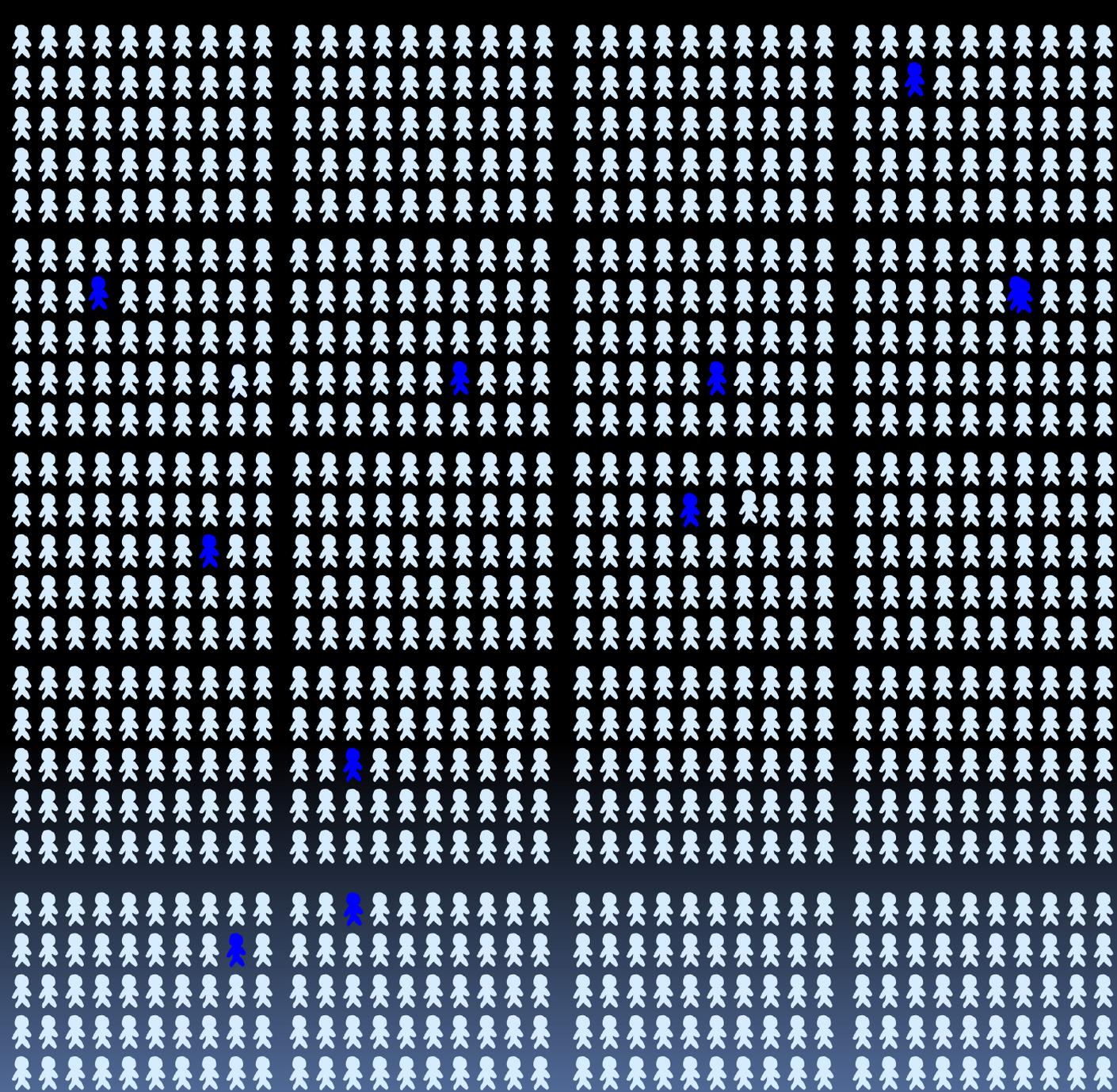


**Fred has size 13**



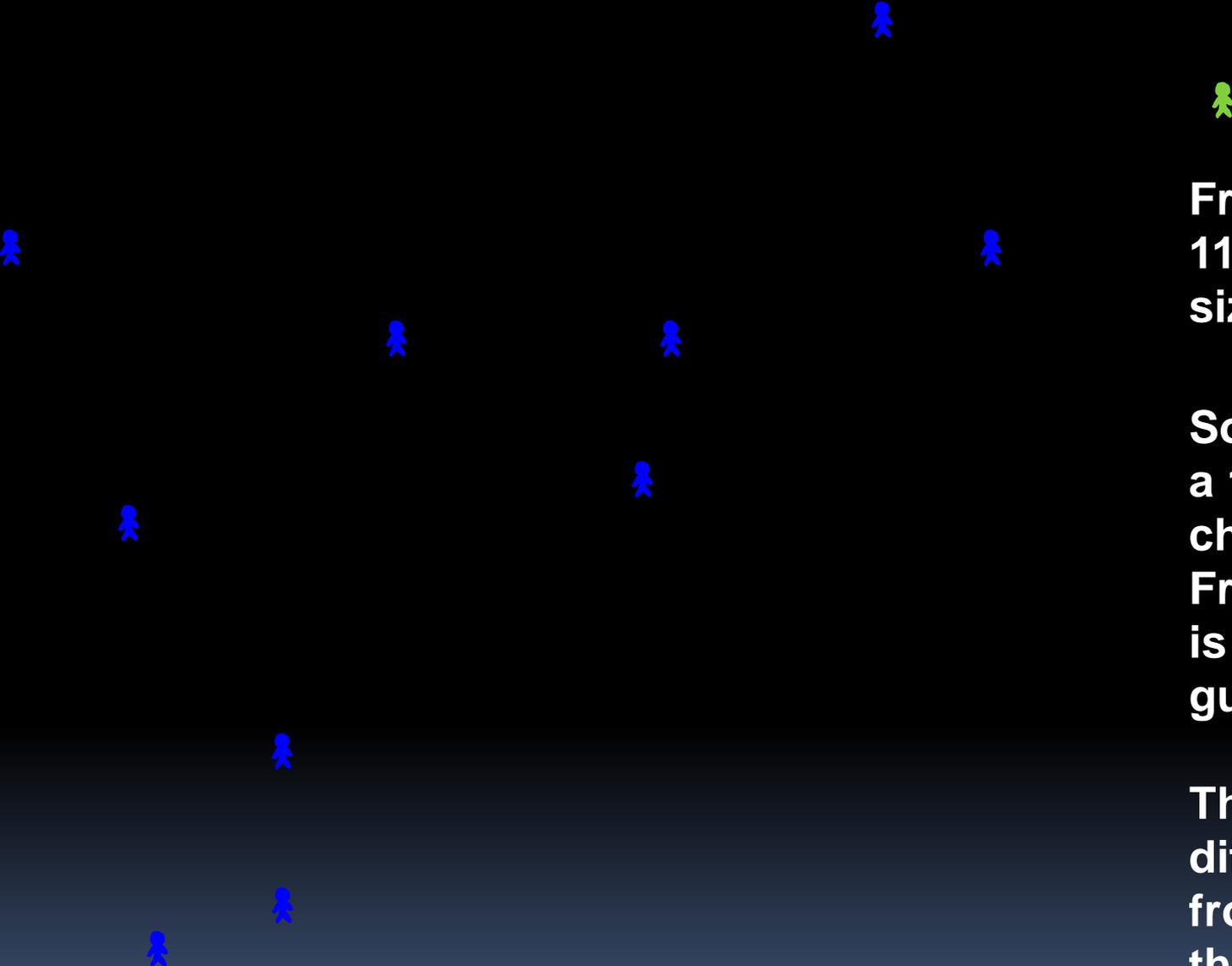
Fred has size 13

Imagine 1,000  
other people  
also at scene



Fred has size 13

About 10  
out of the  
1,000 people  
have size 13

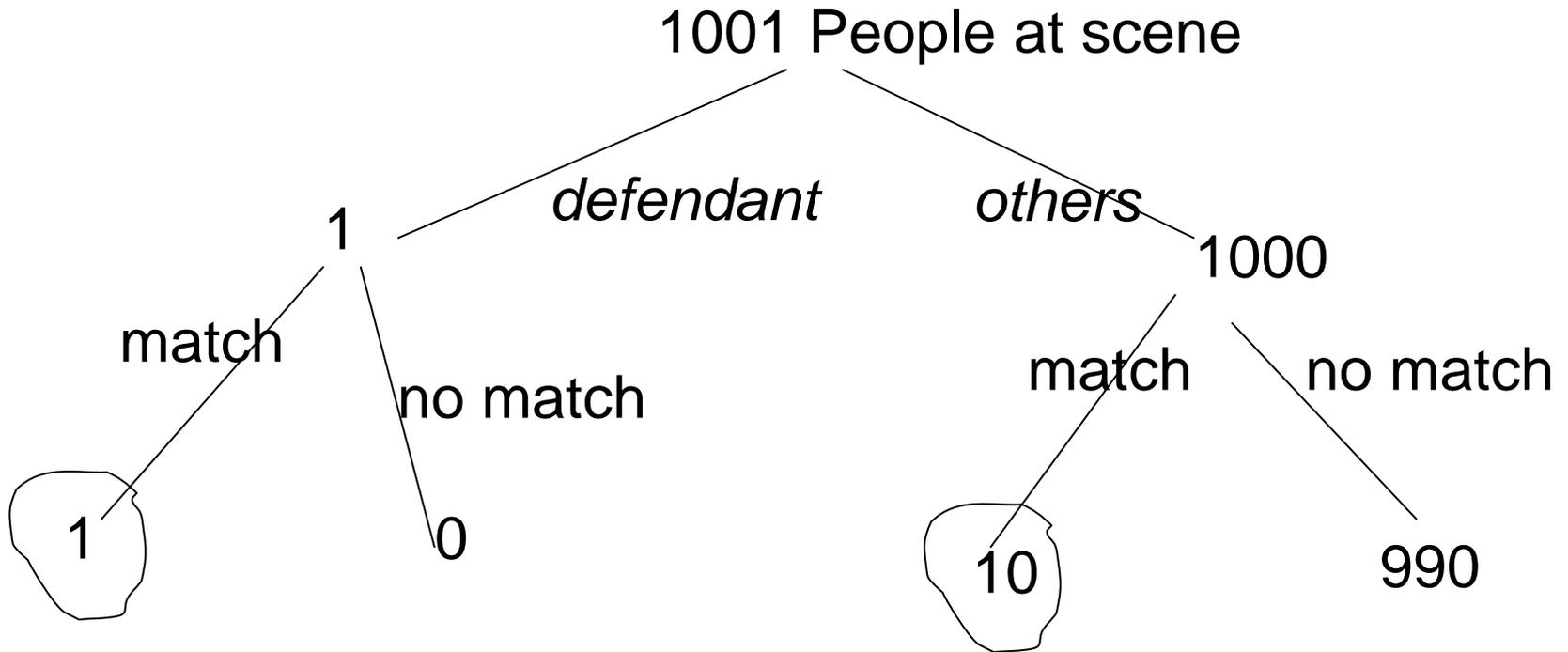


**Fred is one of  
11 with  
size 13**

**So there is  
a 10/11  
chance that  
Fred  
is NOT  
guilty**

**That's very  
different  
from  
the  
prosecution  
claim of 1%**

# Decision Tree Equivalent



30 November 2011 Last updated at 17:48

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## Stephen Lawrence trial: 'Blood on Gary Dobson's jacket'

**A stain on a murder accused's jacket was caused by fresh blood with a one-in-a-billion chance of not being victim Stephen Lawrence's, a court has heard.**

Forensic scientist Edward Jarman said if blood found on Gary Dobson's jacket had been old when it made contact, it would not have soaked in.

He said the blood could have been "shed from a knife" and would have dried in a couple of minutes.

Mr Dobson, 36, and David Norris, 35, deny murdering 18-year-old Mr Lawrence.

The defence says police contaminated evidence relating to the killing.



A close-up of a blood spot found on the jacket police recovered from Gary Dobson's home

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### Related Stories

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[Lawrence case database 'altered'](#)

[Questions over](#)

How the fallacy is also stated

**“The chances of finding this evidence in an innocent man are so small that you can safely disregard the possibility that this man is innocent”**

# Ahh.. but DNA evidence is different?

- Very low random match probabilities ... but same error
- Low template DNA 'matches' have high random match probabilities
- Probability of testing/handling errors not considered
- Principle applies to ALL types of forensic match evidence

# Determining the value of evidence

***Prosecution likelihood*** (The probability of seeing the evidence if the prosecution hypothesis is true)

(=1 in example)

***Defence likelihood*** (The probability of seeing the evidence if the defence hypothesis is true)

(=1/100 in example)

**Likelihood ratio =  $\frac{\text{Prosecutor likelihood}}{\text{Defence likelihood}}$**  (=100 in example)

Providing hypotheses are “guilty” and “not guilty”

LR > 1 supports prosecution;

LR <1 supports defence

LR = 1 means evidence has no probative value

# Bayes Theorem (“Odds Form”)

**Posterior Odds = Likelihood ratio x Prior Odds**

**Prior odds      Likelihood ratio      Posterior Odds**

<b>Prosecutor</b>	<b>1</b>		<b>100</b>		<b>1</b>
		<b>X</b>		<b>=</b>	
<b>Defence</b>	<b>1000</b>		<b>1</b>		<b>10</b>

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<b>Prosecutor</b>	<b>1</b>		<b>100</b>		<b>25</b>
		<b>X</b>		<b>=</b>	
<b>Defence</b>	<b>4</b>		<b>1</b>		<b>1</b>

# Was Mrs Peacock the murderer?



H<sub>p</sub>: “Mrs Peacock guilty”

E: “The murderer was a woman

$$P(E | H_p) = 1$$

$$P(E | H_d) = 2/5$$

$$LR = 2.5$$

But  
if H<sub>d</sub>: “Miss Scarlet was the murderer”

$$LR = 1$$

# Sally Clark Revisited: A new flaw in the probability experts' reasoning

**Hd : Sally Clark's two babies died of SIDS**

**Hp : Sally Clark murdered her two babies**

**“(Prior) probability of Hd over 100 times greater than (prior) probability of Hp”**

**“So assuming LR of 5 .....**

**Hd : Sally Clark's two babies died of SIDS**

**Hp : Sally Clark murdered at least one of her two babies.**



**(Prior) probability of Hd only 2.5 times greater than the (prior) probability of Hp**

# Tip of the Fallacies Iceberg

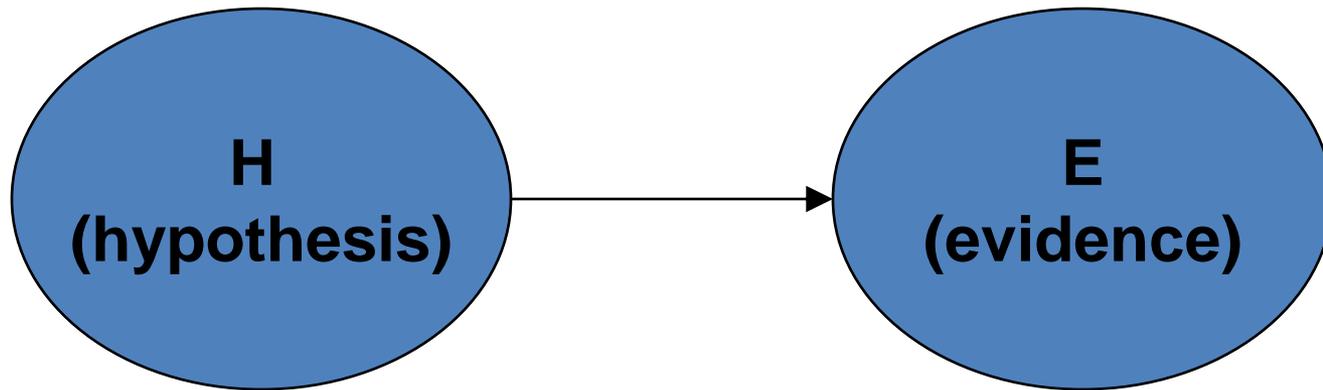
- Defendant fallacy
- Confirmation bias fallacy
- Base rate neglect
- Treating dependent evidence as independent
- Coincidences fallacy
- Various evidence utility fallacies
- Cross admissibility fallacy
- 'Crimewatch UK' fallacy

**Fenton, N.E. and Neil, M., 'Avoiding Legal Fallacies in Practice Using Bayesian Networks', Australian Journal of Legal Philosophy 36, 114-151, 2011**

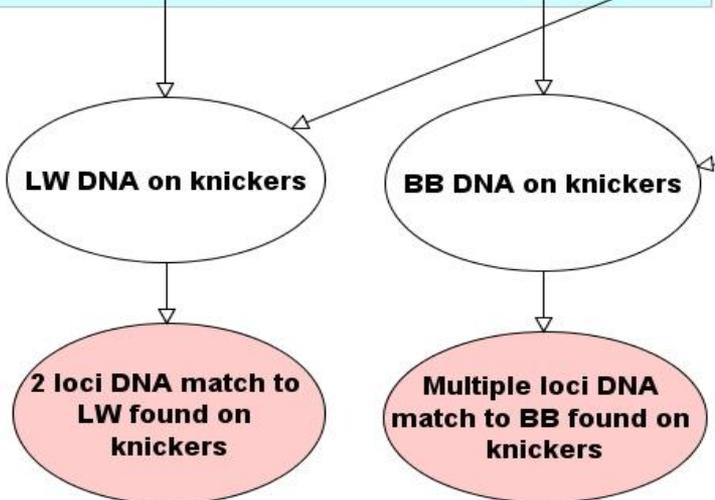
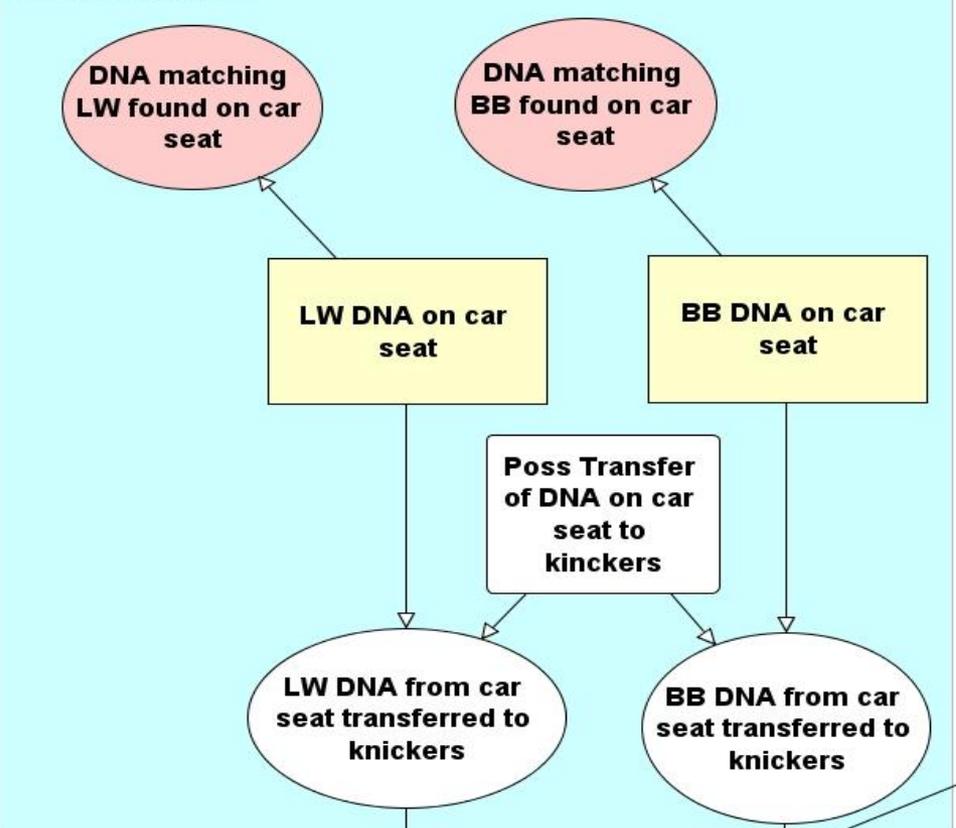
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# **THE SCALING PROBLEM AND OTHER CHALLENGES OF BAYES**

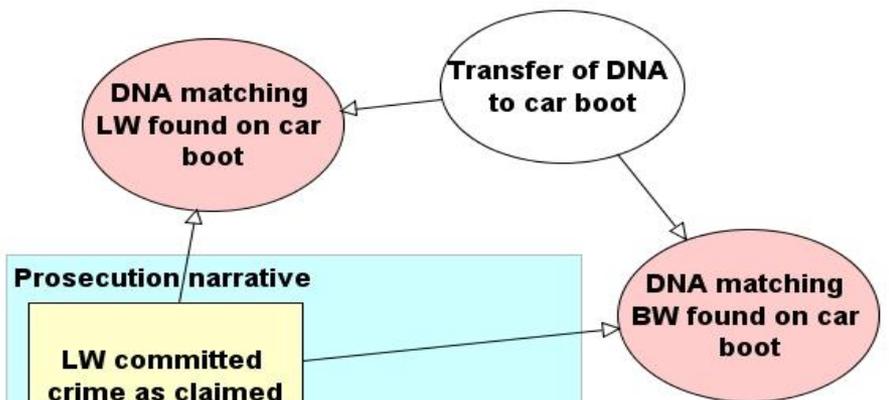
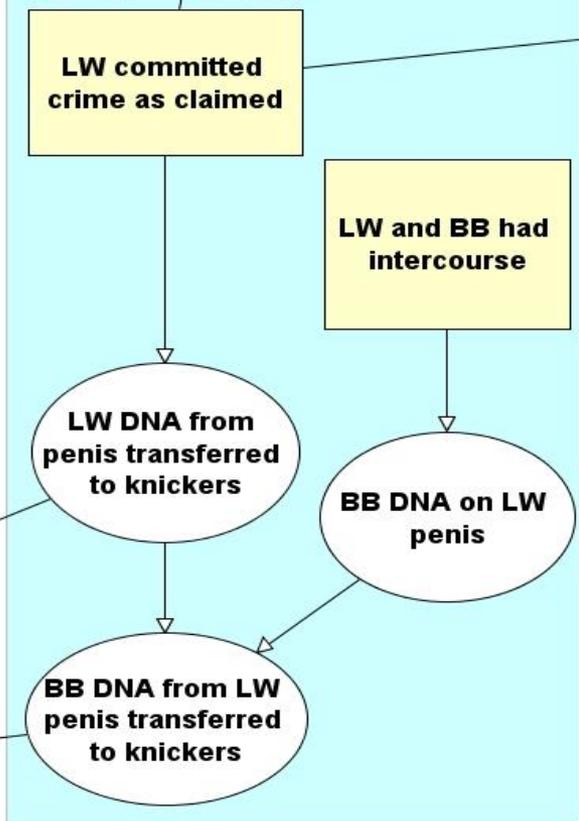
# The basic legal argument



**Defence narrative**

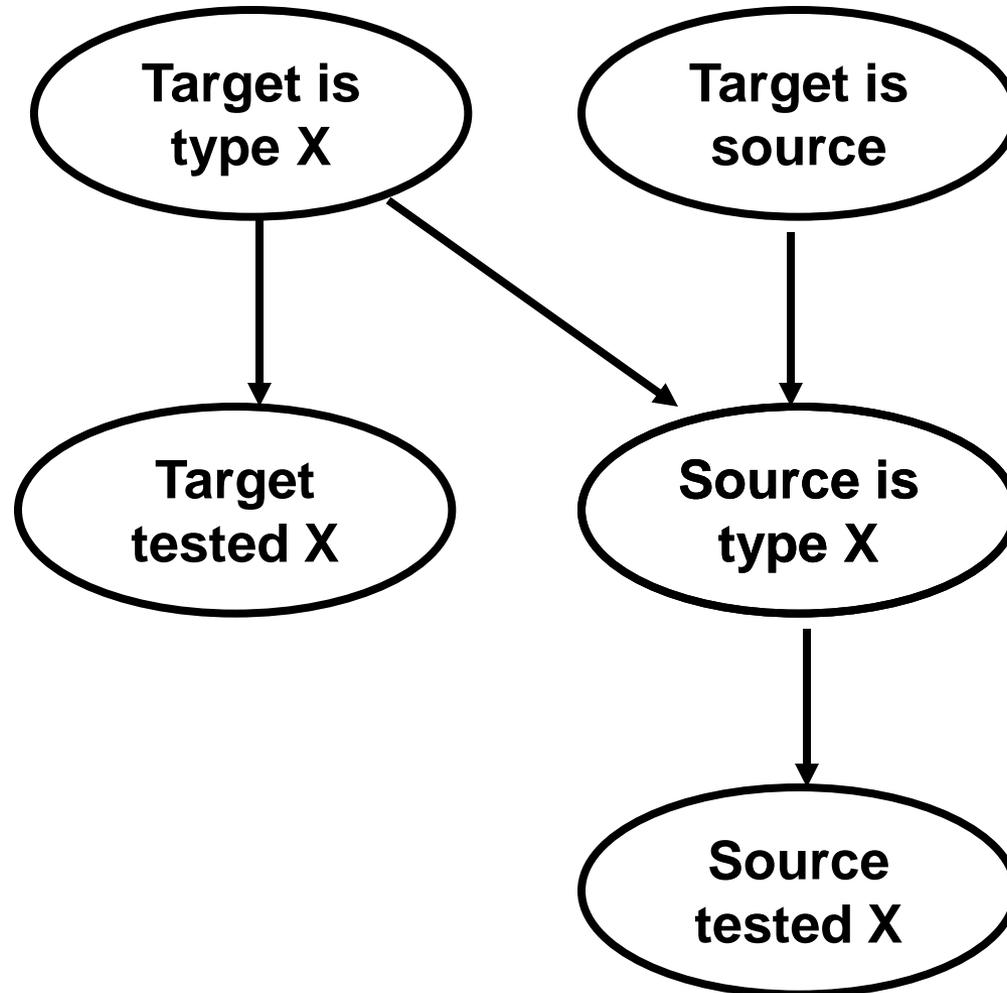


**Prosecution narrative**



**..but this is a typical real BN**

# Even single piece of forensic match evidence is NOT a 2-node BN



# Decision Tree far too complex

H1: target = source

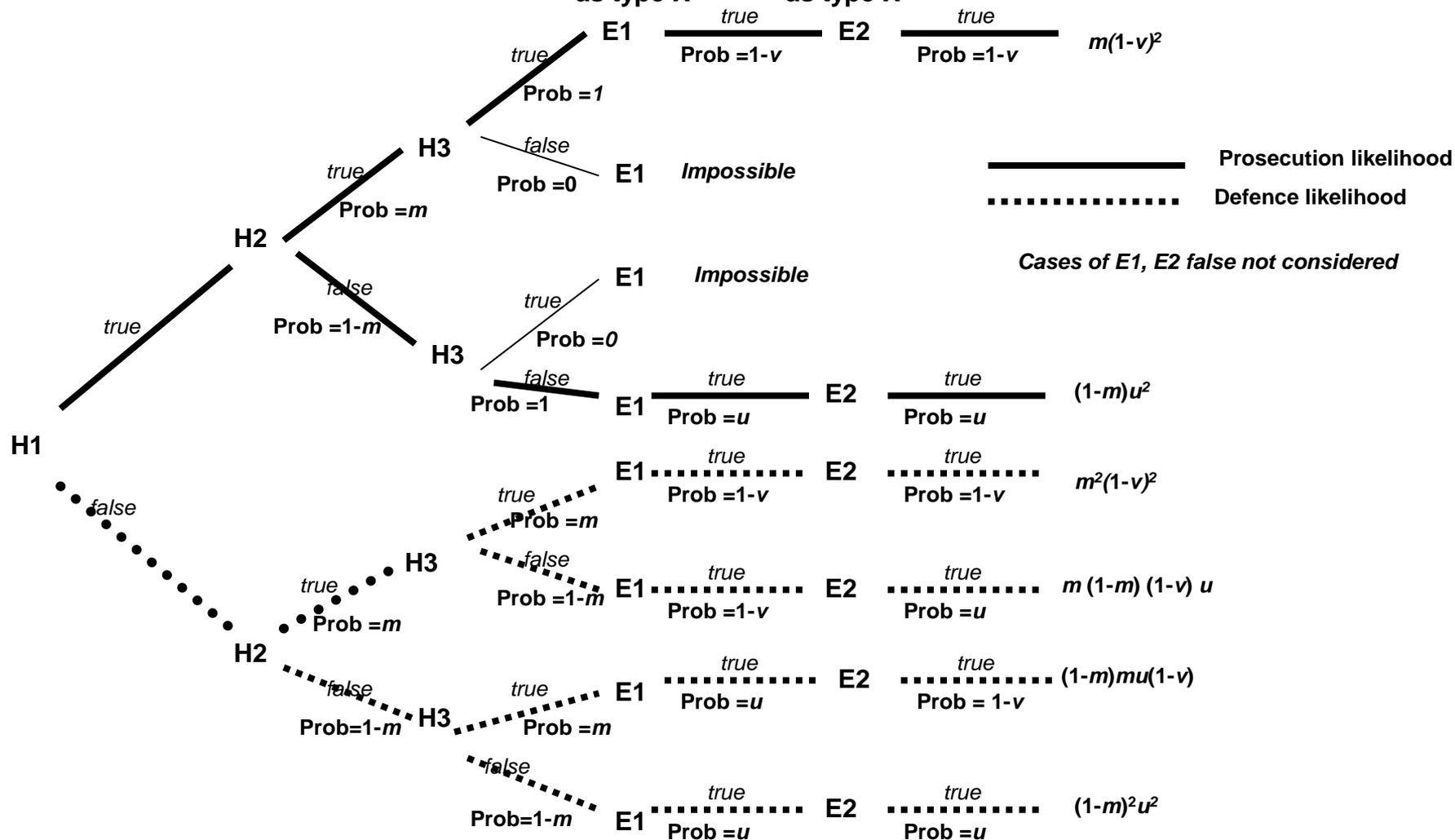
H2: source is type X

H3: target is type X

E1: source tested as type X

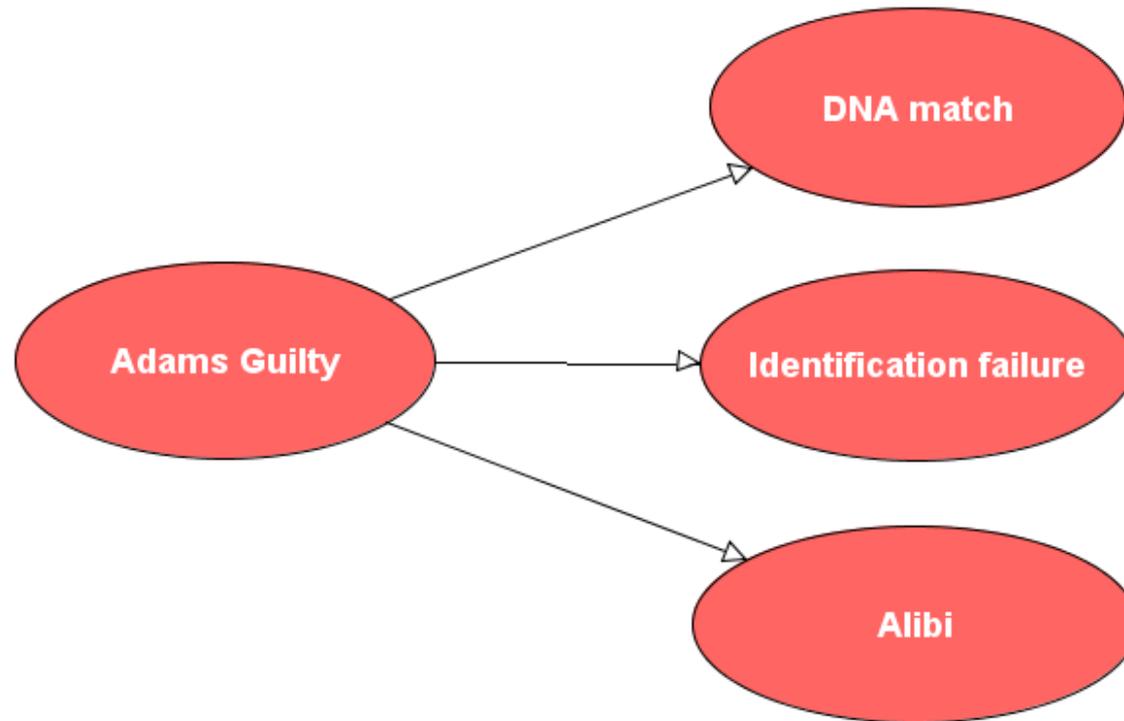
E2: target tested as type X

Probability of branch



$m$  is the random match probability for type X  
 $u$  is the false positive probability for X  
 $v$  is the false negative probability for X

# Even worse: do it formulaically from first principles

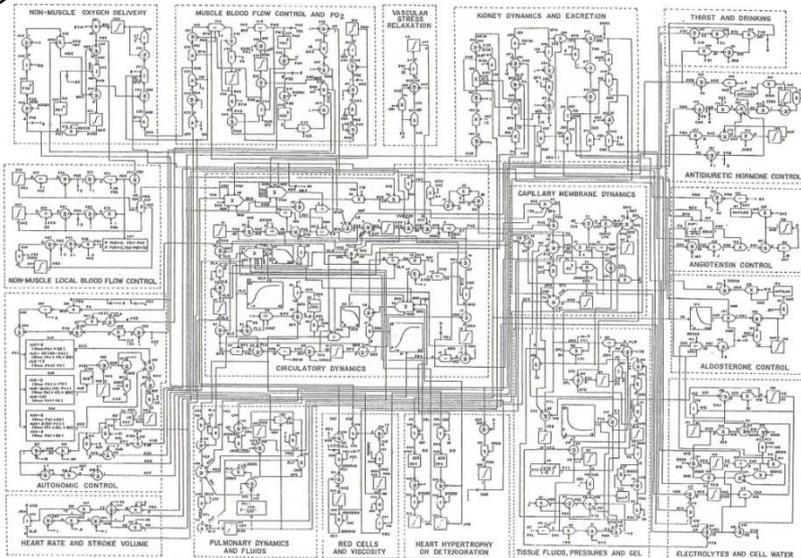
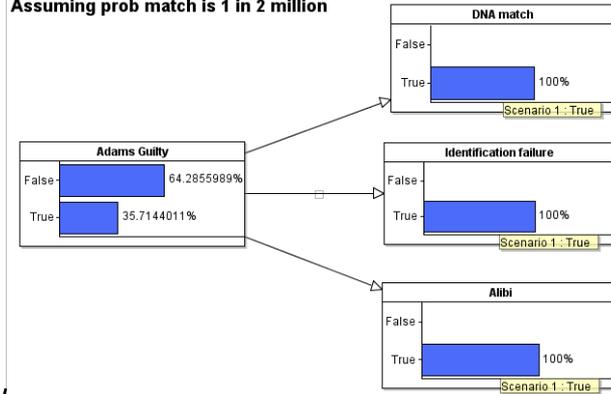


$$\begin{aligned} V &= \frac{\Pr(H_p | E, I_1, I_2)}{\Pr(H_d | E, I_1, I_2)} \\ &= \frac{\Pr(E | H_p)}{\Pr(E | H_d)} \times \frac{\Pr(I_1 | H_p)}{\Pr(I_1 | H_d)} \times \frac{\Pr(I_2 | H_p)}{\Pr(I_2 | H_d)} \times \frac{\Pr(H_p)}{\Pr(H_d)} \end{aligned}$$

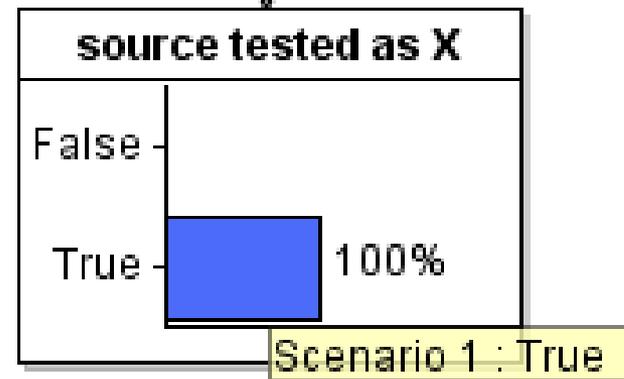
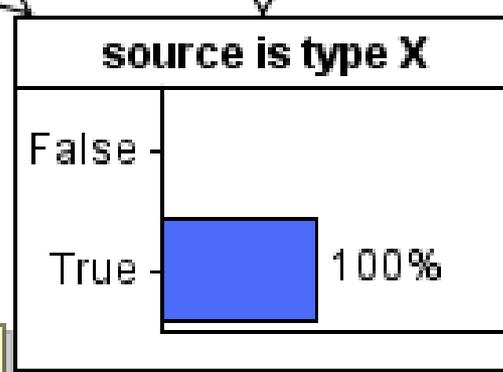
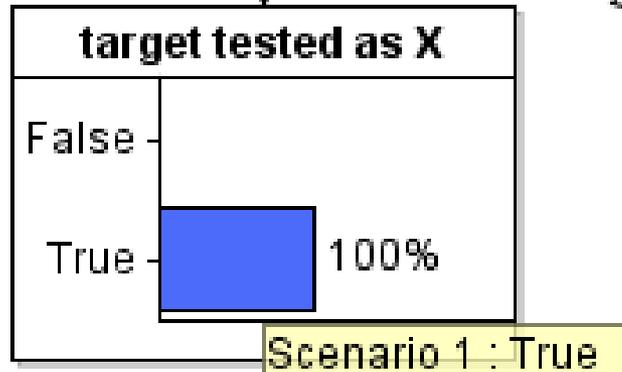
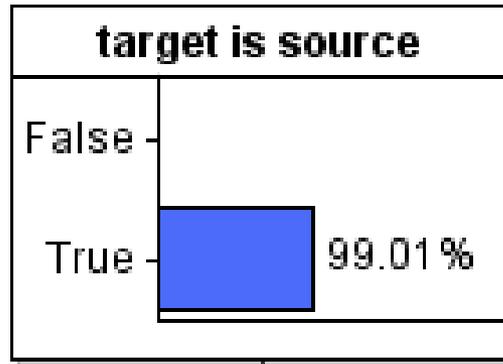
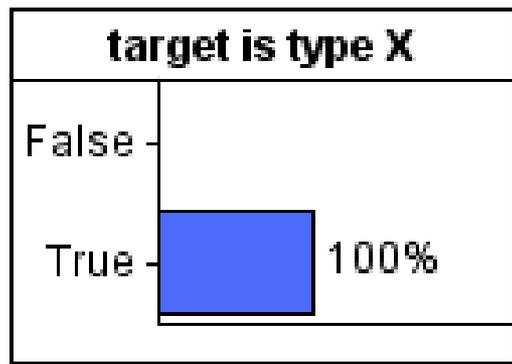
# Hence the Calculator Analogy



Assuming prob match is 1 in 2 million

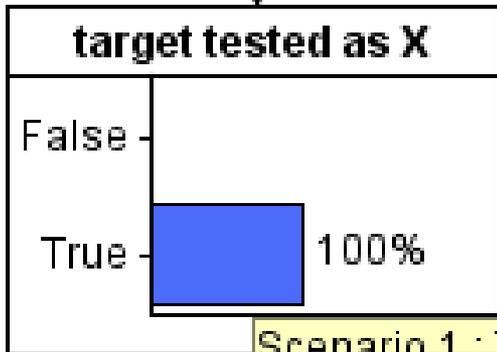
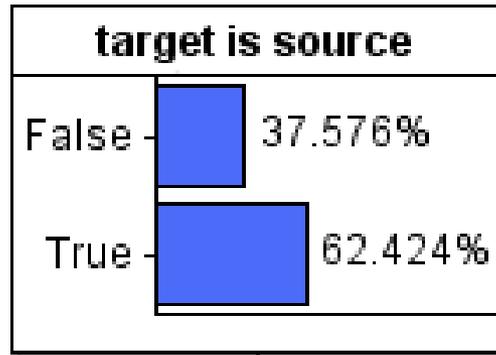
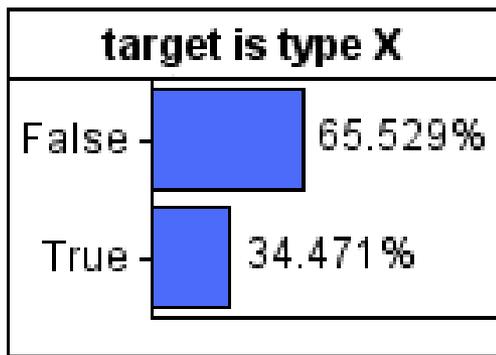


$P(H_p) = P(H_D) = 0.5$   
 $P(E | H_p, H_D) = 0.9$   
 $P(E | H_p, \text{not } H_D) = 0.9$   
 $P(E | \text{not } H_p, H_D) = 0.9$   
 $P(E | \text{not } H_p, \text{not } H_D) = 0$   
 Then  
 $P(E | H_p) = P(E | H_p, H_D)P(H_D) + P(E | H_p, \text{not } H_D)P(\text{not } H_D) = 0.9 \times 0.5 + 0.9 \times 0.5 = 0.9$   
 and  
 $P(E | H_D) = P(E | H_D, H_p)P(H_p) + P(E | H_D, \text{not } H_p)P(\text{not } H_p) = 0.9 \times 0.5 + 0.9 \times 0.5 = 0.9$   
 so  
 $P(E | H_p) = P(E | H_D)$  i.e. the LR is equal to 1  
 Now we can also use marginalisation to compute  $P(E)$ :  
 $P(E) = P(E | H_p, H_D)P(H_p)P(H_D) + P(E | H_p, \text{not } H_D)P(H_p)P(\text{not } H_D) +$   
 $+ P(E | \text{not } H_p, H_D)P(\text{not } H_p)P(H_D) + P(E | \text{not } H_p, \text{not } H_D)P(\text{not } H_p)P(\text{not } H_D)$   
 $= (0.9 \times 0.5 \times 0.5) + (0.9 \times 0.5 \times 0.5) + (0.9 \times 0.5 \times 0.5) + 0$   
 $= 0.675$   
 Hence by Bayes:  
 $P(H_p | E) = \frac{P(E | H_p)P(H_p)}{P(E)} = \frac{0.9 \times 0.5}{0.675} = 0.666$   
 Similarly:  
 $P(H_D | E) = \frac{P(E | H_D)P(H_D)}{P(E)} = \frac{0.9 \times 0.5}{0.675} = 0.666$

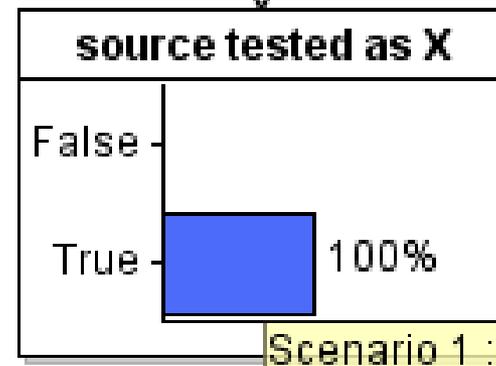
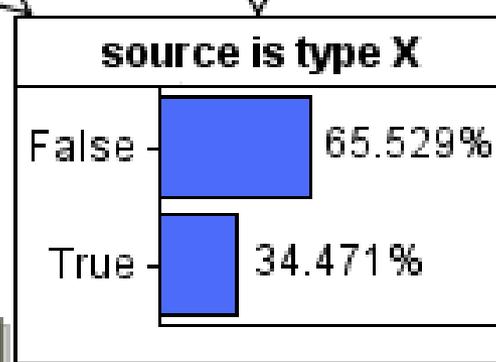


Assumes perfect test accuracy

(this is a 1/1000 random match probability)



Scenario 1 : True



Scenario 1 : True

Assumes

false  
positive  
rate 0.1

false  
negative  
rate 0.01

# The Classic Challenges

“No such thing as probability”

“Cannot combine ‘subjective’ evidence with ‘objective’ (the DNA obsession)”

Defining subjective priors

Getting consensus from other Bayesians

# Bayesian nets: what we need to stress

Separate out assumptions from calculations

Can incorporate subjective, expert judgement

Can address the standard resistance to using subjective probabilities by using *ranges*.

Easily show results from different assumptions

...but must be seen as the 'calculator'

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# **BNS APPLIED TO THE LEGAL CASES**

# R v Barry George (revisiting the Appeal Court judgment)

H: Hypothesis “Barry George did not fire gun”

E: Particle of FDR in coat pocket

Defence likelihood  $P(E|H) = 1/100$

...

But Prosecution likelihood  $P(E| \text{not } H) = 1/100$

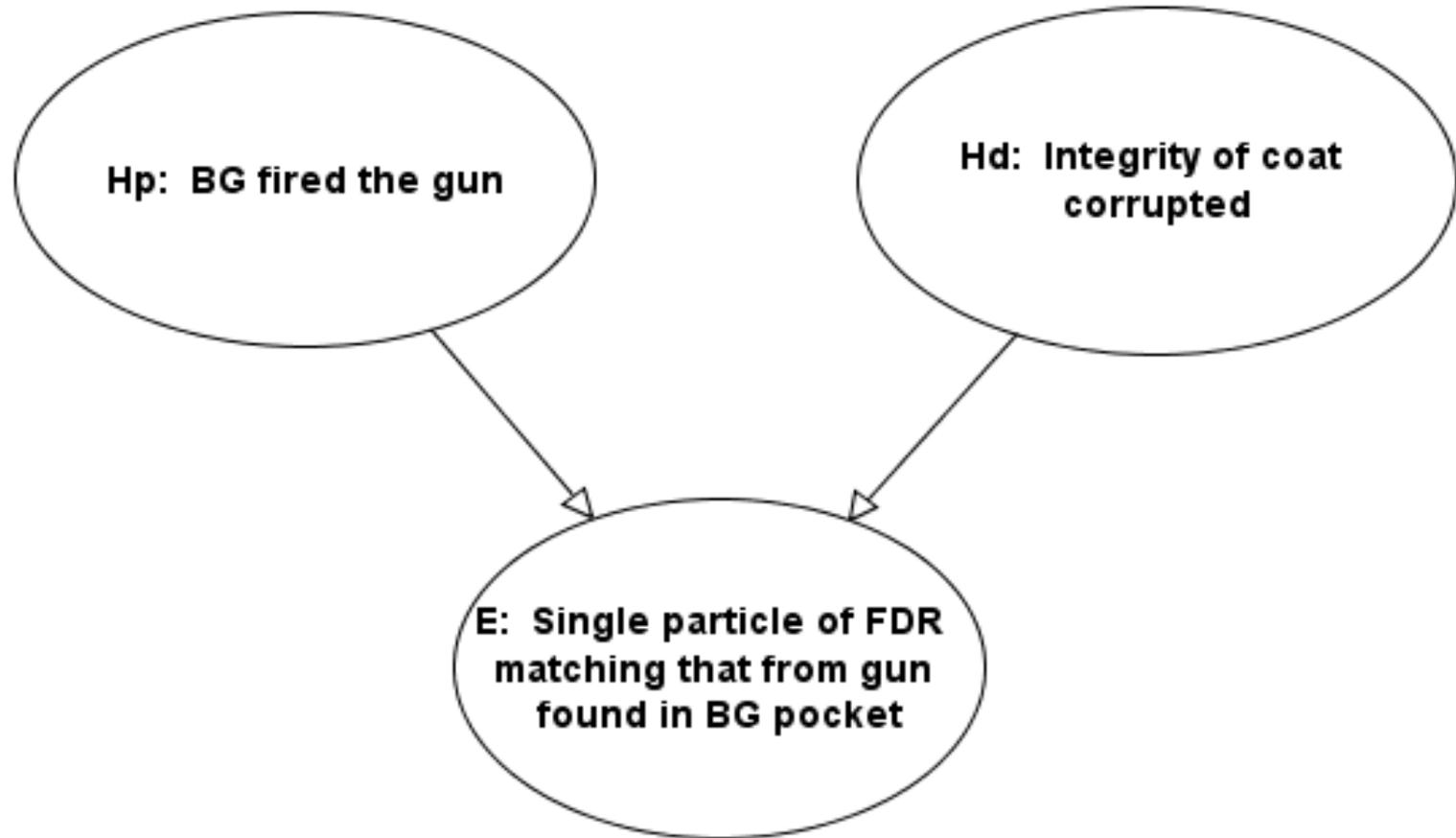
So LR = 1 and evidence ‘has no probative value’

But the argument is fundamentally flawed



Fenton, N. E., D. Berger, D. Lagnado, M. Neil and A. Hsu, (2013). "When ‘neutral’ evidence still has probative value (with implications from the Barry George Case)", *Science and Justice*, <http://dx.doi.org/10.1016/j.scijus.2013.07.002>

**LR=1 but hypotheses not mutually exclusive and exhaustive**



**.....E has real probative value on Hp**

# R v Bellfield

Numberplate evidence

Prosecution opening fallacies

Judge's instruction to Prosecuting QC



**... but on 12 Feb 2008:  
"Forensic scientist Julie-Ann  
Cornelius told the court the  
chances of DNA found on Sally  
Anne's body not being from  
Dixie were a billion to one."**



# R v Dobson

Probabilistic flaws in forensic reports

Revealed in cross-examination of experts

Newspaper reported fallacies *wrongly reported*



5

# **CONCLUSIONS AND WAY FORWARD**

# Misplaced optimism?

**“I assert that we now have a technology that is ready for use, not just by the scholars of evidence, but by trial lawyers.”**

**Edwards, W. (1991). "Influence Diagrams, Bayesian Imperialism, and the Collins case: an appeal to reason." Cardozo Law Review 13: 1025-107**

# Summary

**Correct probability reasoning is central to far more cases than people imagine**

**Errors of reasoning plague the system**

**Sometimes Bayesian experts compound the problem**

**Doing things correctly requires BNs**

**But Bayesian arguments cannot be presented from first principles.**

**Focus on the prior assumptions NOT the Bayesian calculations (the calculator analogy)**

# Blatant Plug for New Book



**CRC Press, ISBN: 9781439809105 , ISBN 10: 1439809100**

# A Call to Arms

## Bayes and the Law Network

Transforming Legal Reasoning through Effective use of  
Probability and Bayes

<https://sites.google.com/site/bayeslegal/>

Contact: [norman@eecs.qmul.ac.uk](mailto:norman@eecs.qmul.ac.uk)

- Fenton, N.E. and Neil, M., 'Avoiding Legal Fallacies in Practice Using Bayesian Networks', Australian Journal of Legal Philosophy 36, 114-151, 2011
- Fenton, N. E. (2011). "Science and law: Improve statistics in court." Nature 479: 36-37.
- Fenton, N. E., Neil, M., & Hsu, A. (2013). "Calculating and understanding the value of any type of match evidence when there are potential testing errors". Artificial Intelligence and Law, to appear.
- Fenton, N. E., D. Lagnado and M. Neil (2012). "A General Structure for Legal Arguments Using Bayesian Networks." to appear Cognitive Science.

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