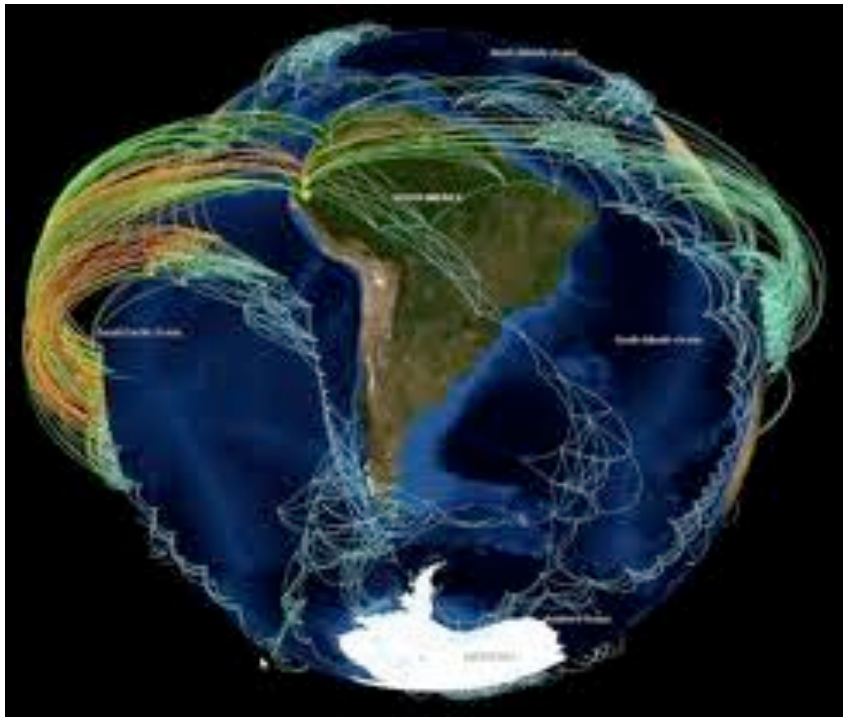
# Complex networks: a useful tool in climate research?



Henk A. Dijkstra
IMAU
Department of Physics & Astronomy,
Utrecht University, NL

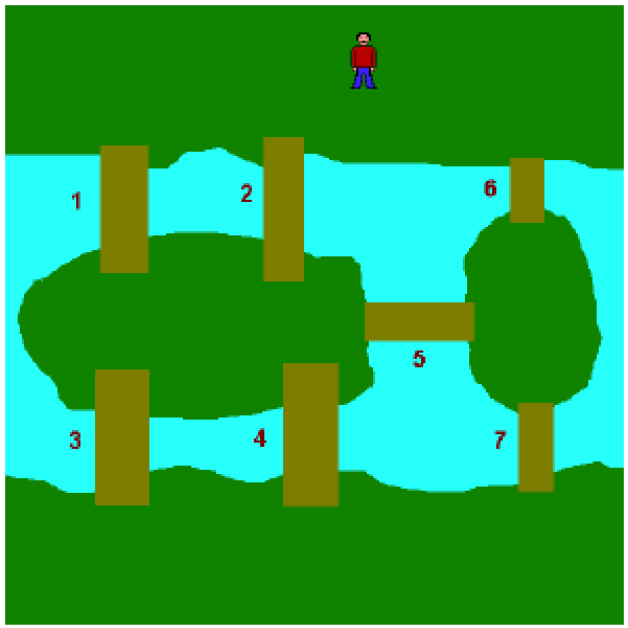
MARIE CURIE ACTIONS

SEVENTH FRAMEWORK PROGRAMME

"LINC"
Learning about Interacting Networks in Climate

# Introduction: Graph Theory

1735: Konigsberg (today: Kaliningrad)
bridge problem:
Can one walk all 7 bridges without crossing a bridge twice?



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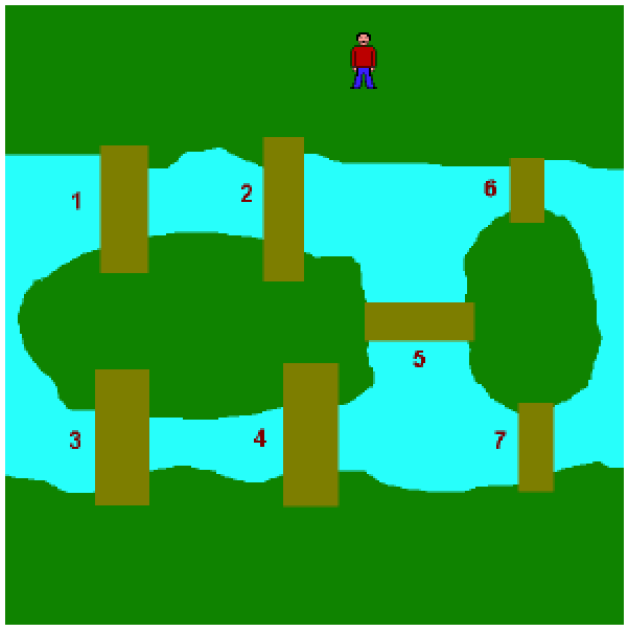


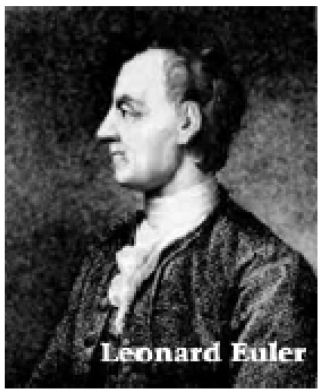


No!

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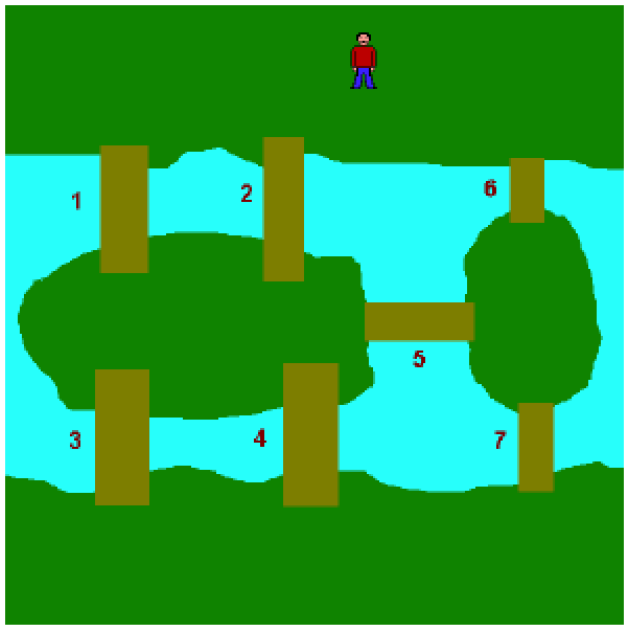


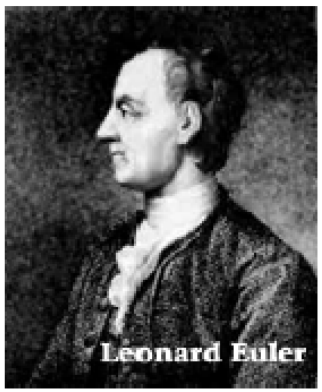


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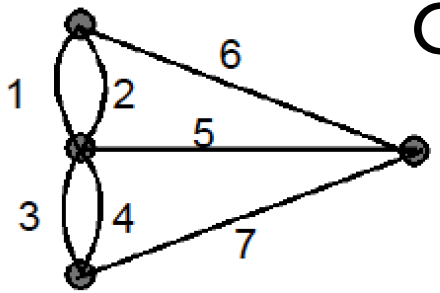
Graph



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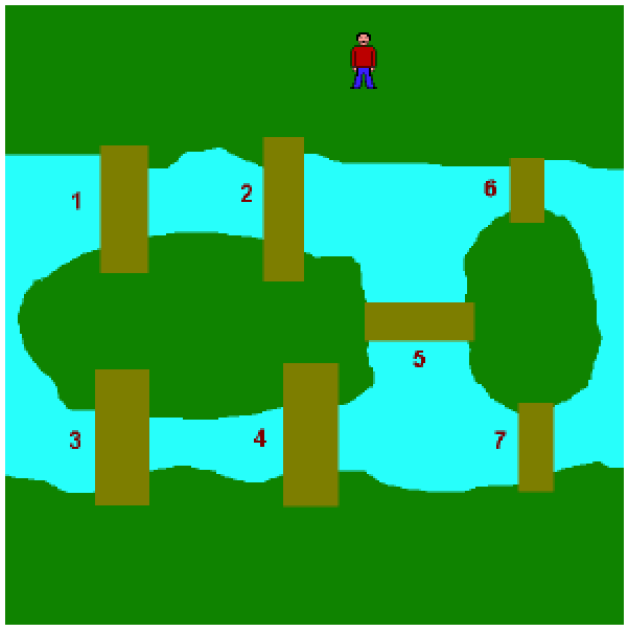


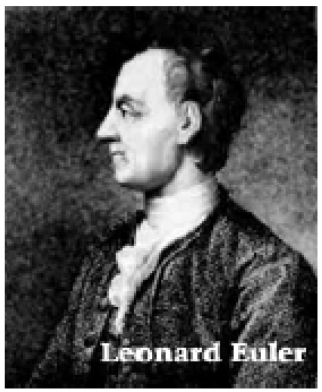


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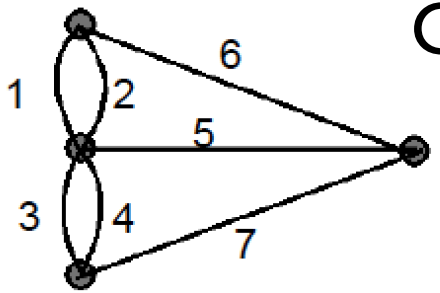
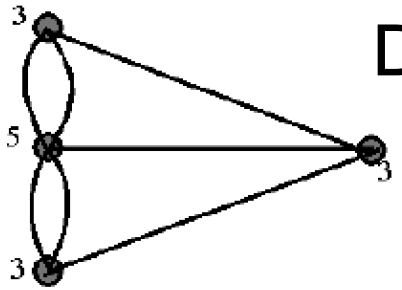



Degree



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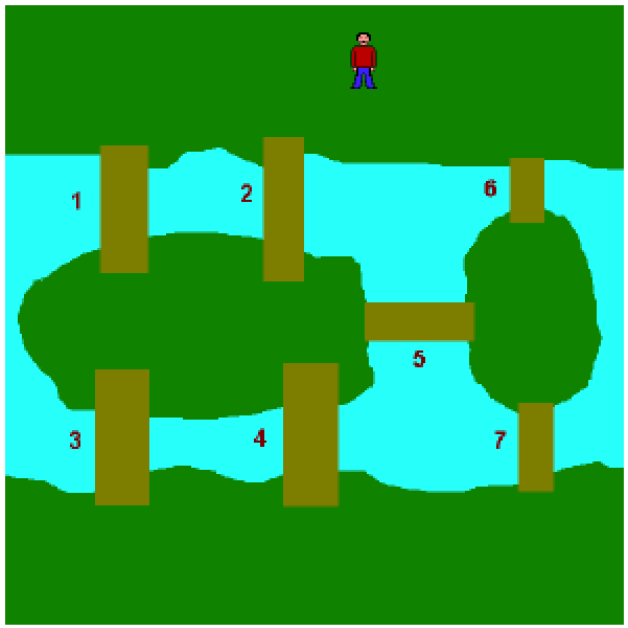


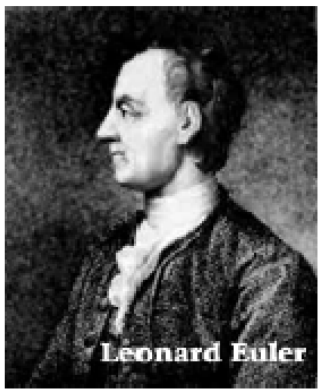


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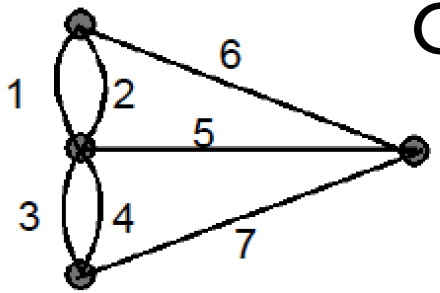


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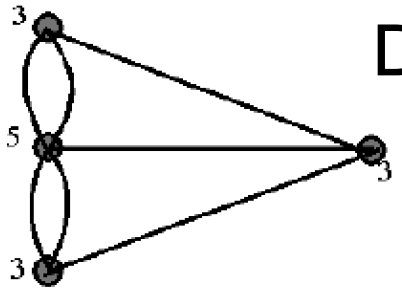


Only possible when there are at most two nodes with an odd degree!

# Graph Theory: Adjacency Matrix

A: adjacency matrix

$A_{ij} = 1$ if node i is connected to node j

$A_{ij} = 0$ otherwise

Graph: G = (V,E)

V: nodes

E: edges

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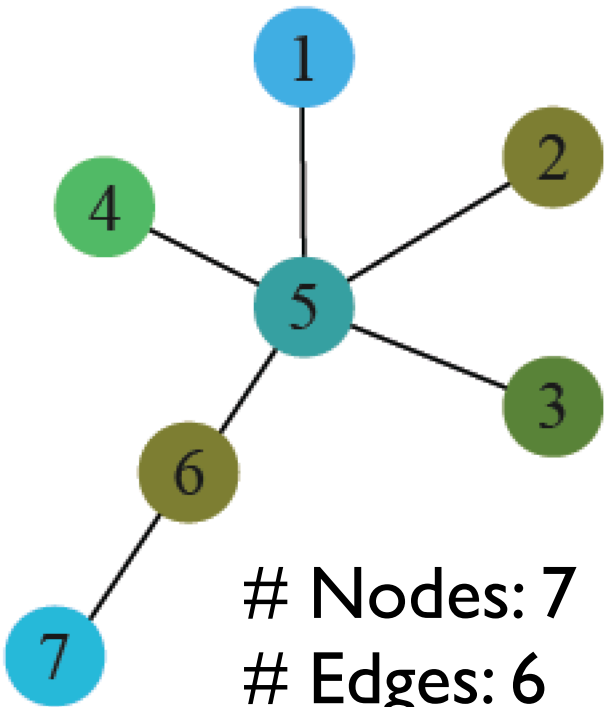
Example: undirected graph




# Nodes: 7

# Edges: 6

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Graph: G = (V,E)

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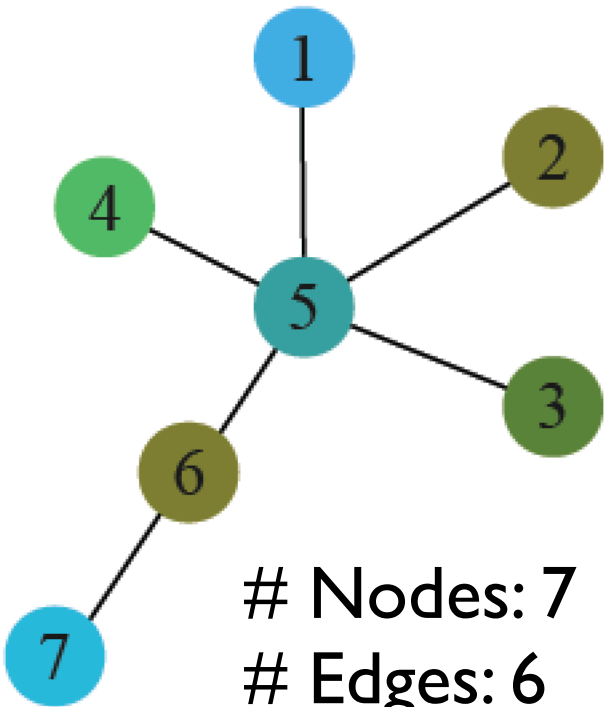
Example: undirected graph

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$k_i = \sum_j A_{ij}$$

Degree of node i




# Nodes: 7

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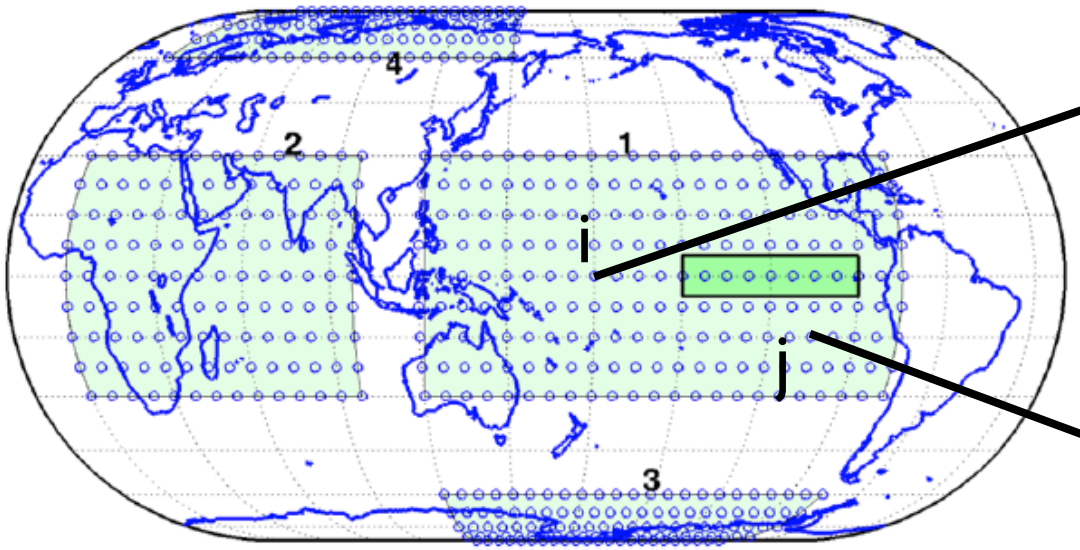
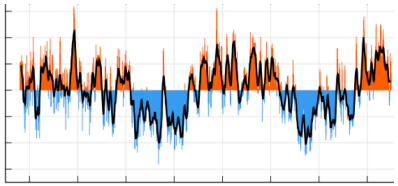
# Climate Networks: Reconstruction

## 1. Interaction Network: correlation



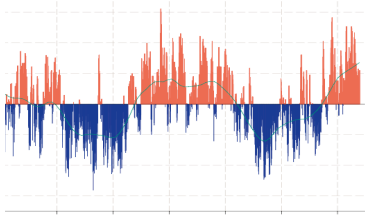

X



t

node i

Y



t

node j

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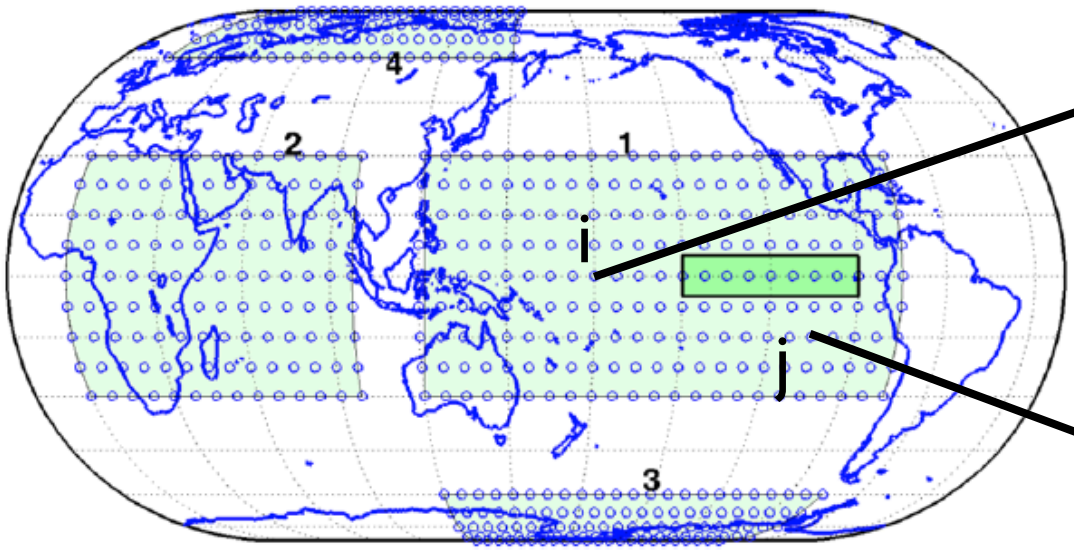
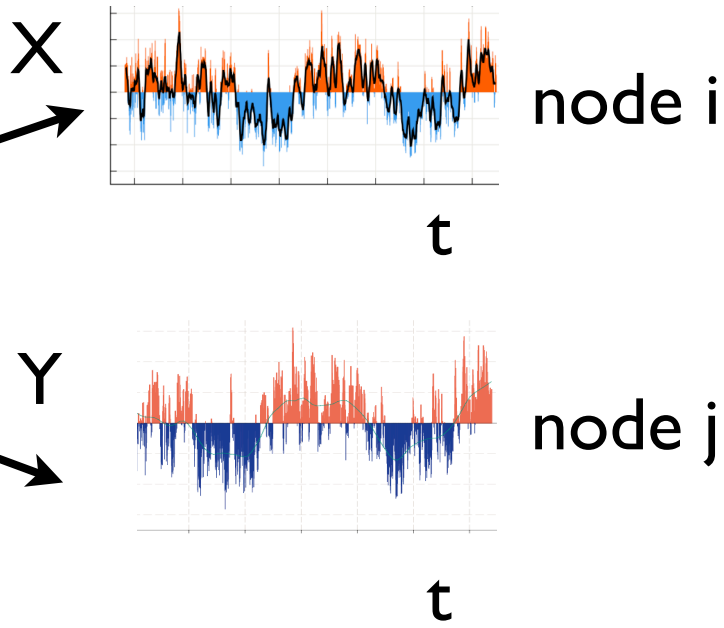



Measure of Correlation

$C_{ij}$

$$r = \frac{\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^{n}(X_i - \bar{X})^2}\sqrt{\sum_{i=1}^{n}(Y_i - \bar{Y})^2}}$$

Pearson correlation

$$I(X;Y) = \sum_{y \in Y}\sum_{x \in X} p(x,y)\log\left(\frac{p(x,y)}{p(x)\,p(y)}\right)$$

Mutual Information

# Climate Networks: Reconstruction

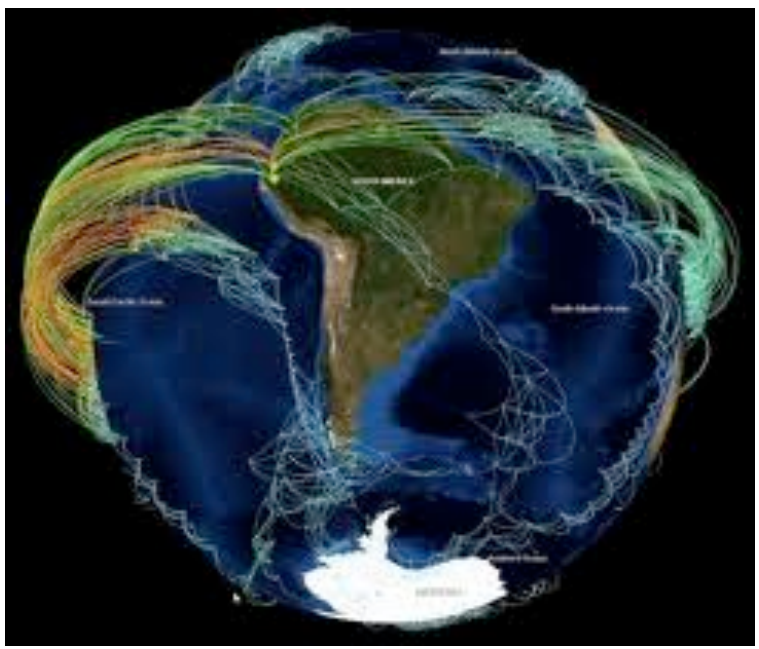
## 1. Interaction Network: construction

$$A_{ij} = \Theta\left(C_{ij} - \tau\right) - \delta_{ij}$$

Heaviside — Kronecker

threshold

*Threshold based on significance analysis!*



Tsonis & Roebber, BAMS, (2004)

Donges et al, EPL, (2009)

# Climate Networks: Reconstruction

## II. Volume Flux Network

$$\begin{aligned} \mathbf{u} &\equiv (u, v) \\ &= (-\partial_y \psi, \partial_x \psi) \end{aligned}$$

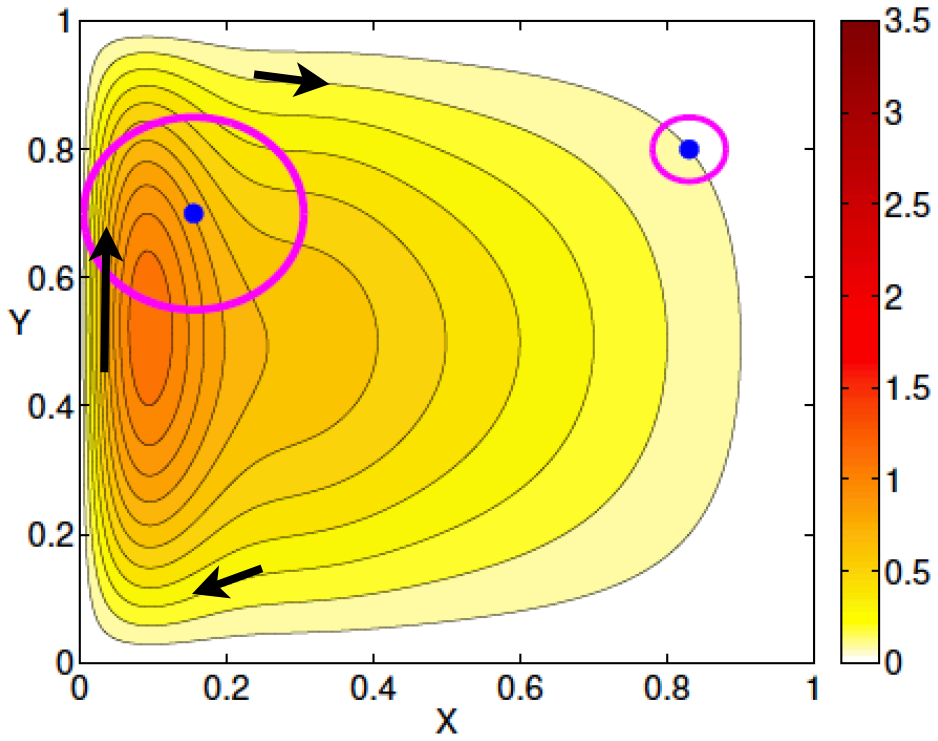
Direction:

$$\underline{\nabla}\psi / |\nabla \psi|$$

Magnitude:

$$\|\mathbf{u}\| = \sqrt{(\partial_x \psi)^2 + (\partial_y \psi)^2}$$

## Streamfunction




Viebahn & Dijkstra, IJBC, subm. (2013)

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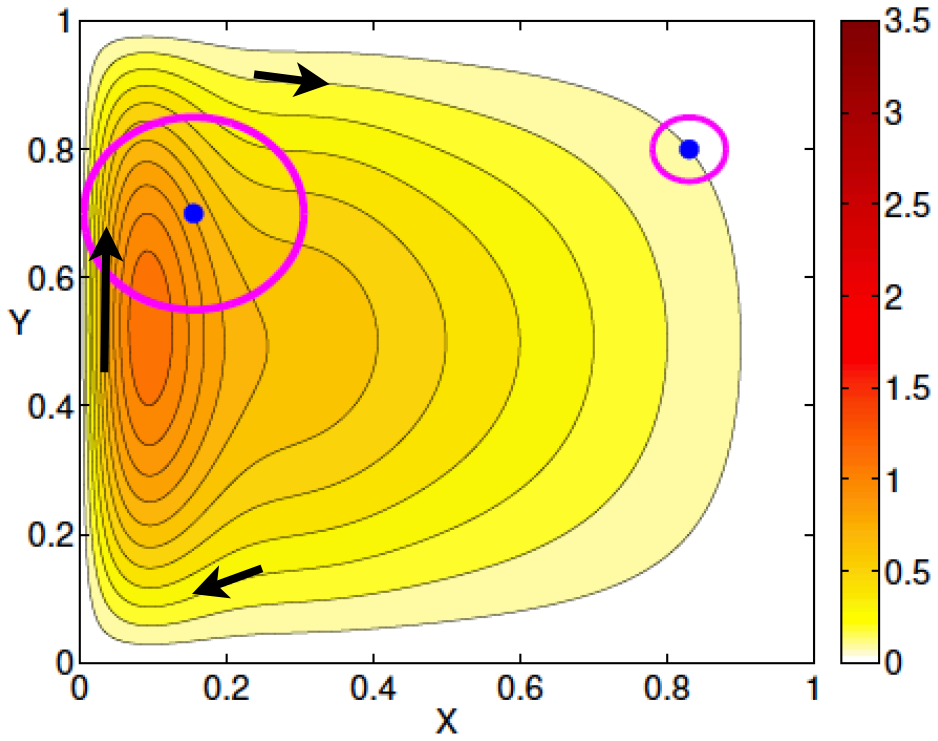



Each node $n \in \mathcal{N}$ links to all the other nodes which are 'related' to the volumetric flux $\mathbf{u}$ through the node under consideration.

**Local link definition:**

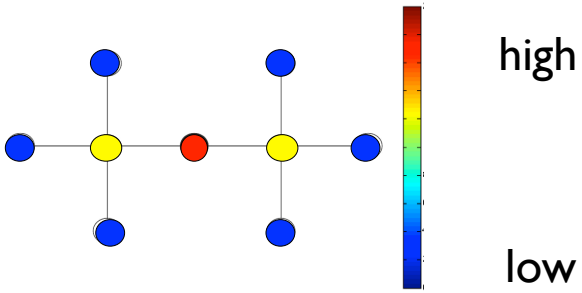
$$N_i = \left\{ n_j \in \mathcal{N} : \|n_i - n_j\| \leq \sqrt{\frac{L}{\pi}} \|\mathbf{u}\| \right\}$$

Viebahn & Dijkstra, IJBC, subm. (2013)

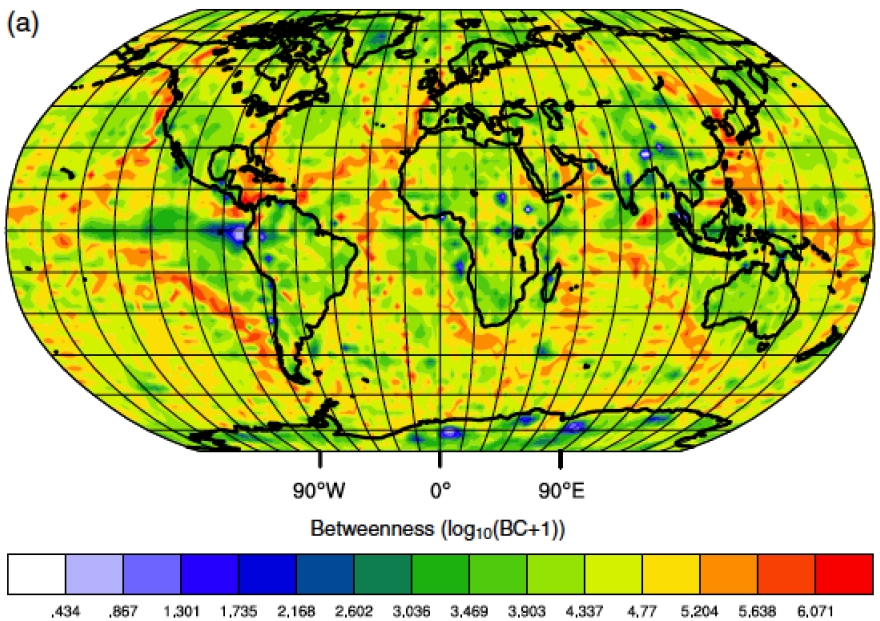
# Climate Networks: Analysis

## NCEP (1948-2007): SAT, MI-network

Betweenness:




Signatures of ocean surface currents??



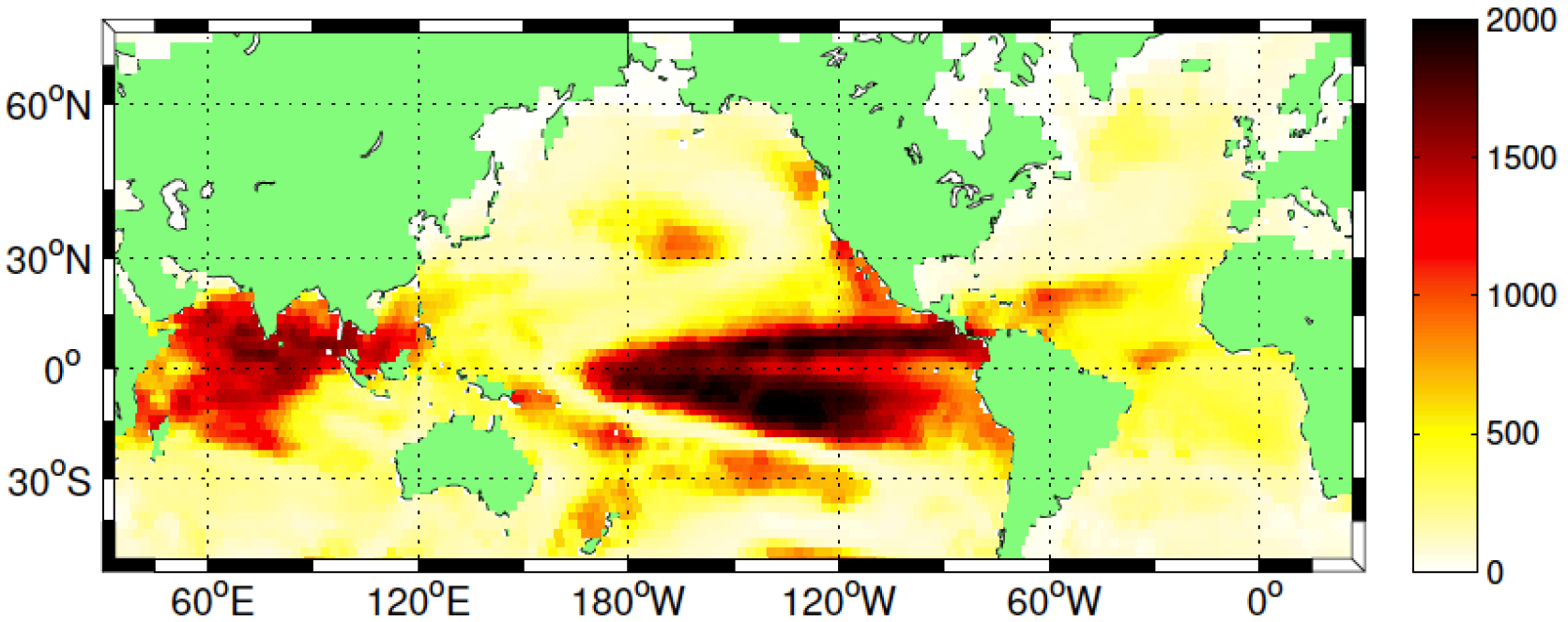

Donges et al, EPL, (2009)

# Climate Networks: Analysis

HadISST (1870-2011): SAT, PC-network

7107 nodes 1704 months

degree distribution



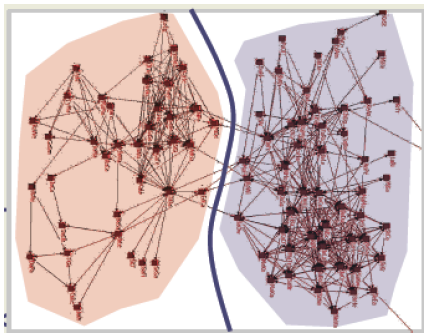

Tantet & Dijkstra, ESD, submitted, (2013)

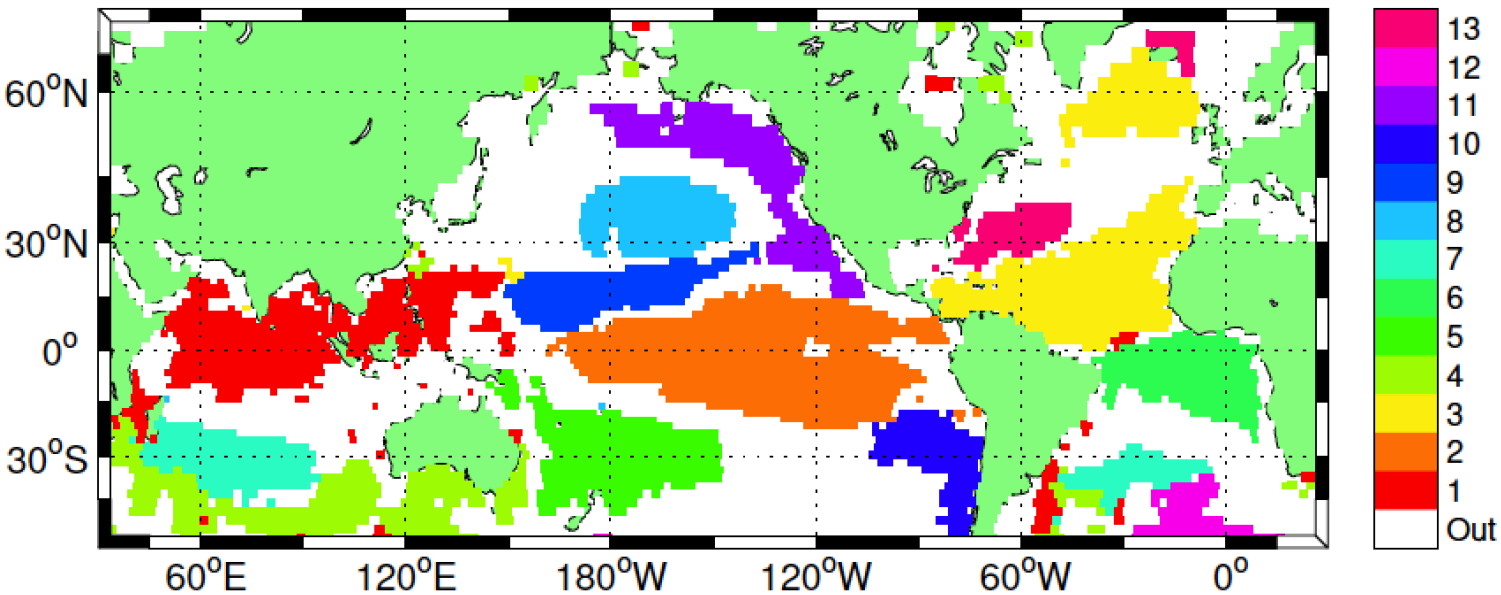
# Climate Networks: Analysis

HadISST (1870-2011): SAT, PC-network

8-year low pass filter

Communities:



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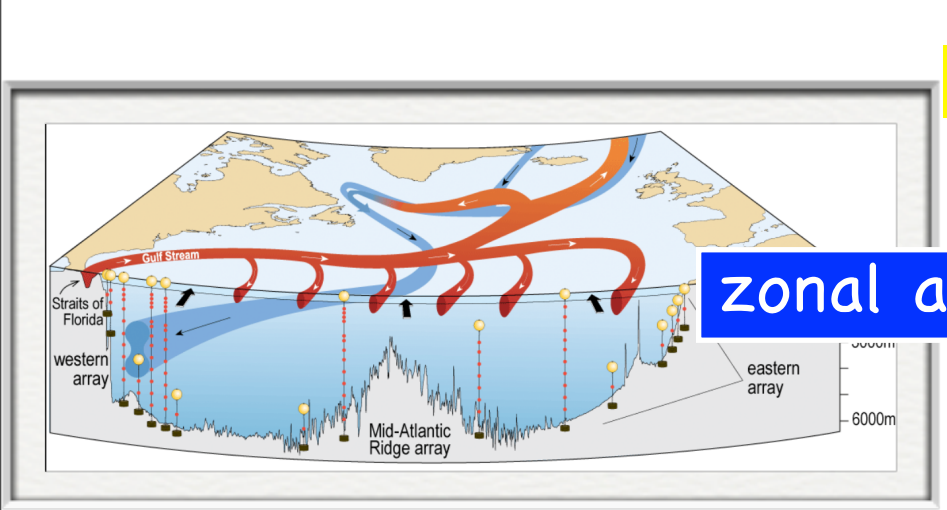
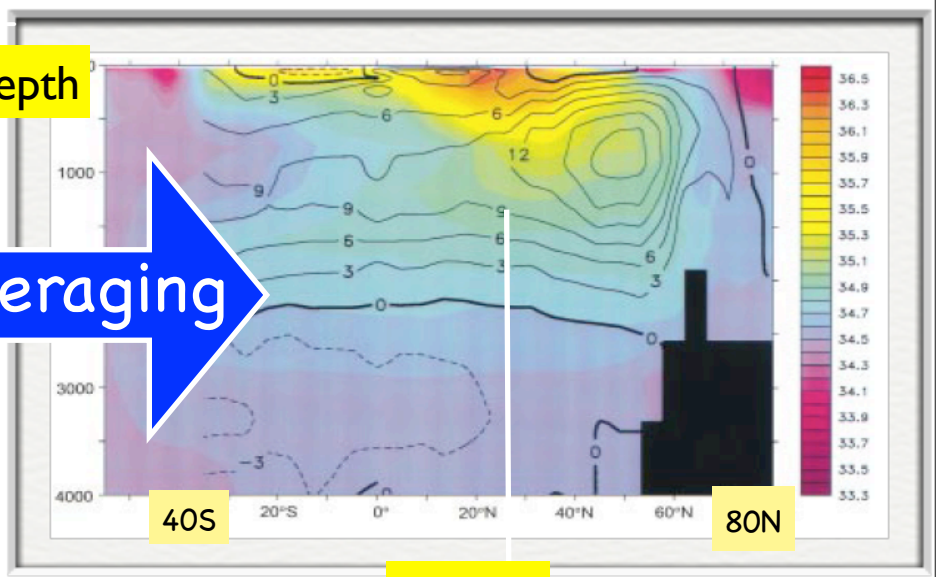
# Application: Interaction Network

Example: transitions in the Atlantic meridional overturning circulation (MOC)



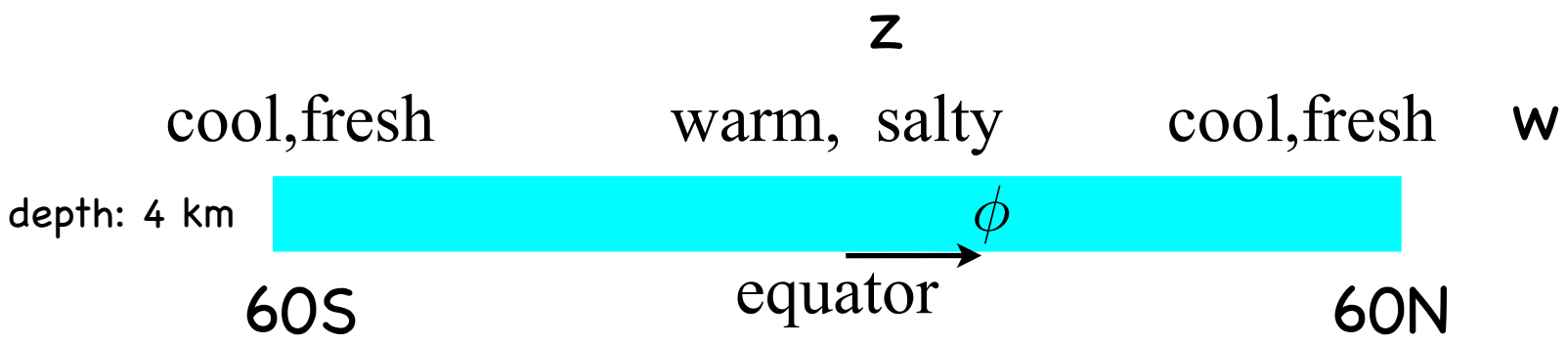

zonal averaging




Main idea: Exploring changes in spatial correlations in the state vector when approaching a bifurcation point

$1\ Sv = 10^6\ \ m^3 s^{-1}$

# Model Problem




equations:

$$\begin{aligned}
\frac{1}{\rho_0 r_0}\frac{\partial p}{\partial \phi} - A_V\frac{\partial^2 v}{\partial z^2} &= \frac{A_H}{r_0^2}\left(\frac{1}{\cos\phi}\frac{\partial}{\partial\phi}\left(\cos\phi\frac{\partial v}{\partial\phi}\right) + \left(1-\tan^2\phi\right)v\right), \\
\frac{1}{\rho_0}\frac{\partial p}{\partial z} &= g\left(\alpha_T T - \alpha_S S\right), \\
0 &= \frac{1}{r_0\cos\phi}\frac{\partial v\cos\phi}{\partial\phi} + \frac{\partial w}{\partial z}, \\
\frac{\partial T}{\partial t} + \frac{v}{r_0}\frac{\partial T}{\partial\phi} + w\frac{\partial T}{\partial z} &= \frac{K_H}{r_0^2\cos\phi}\frac{\partial}{\partial\phi}\left(\cos\phi\frac{\partial T}{\partial\phi}\right) + K_V\frac{\partial^2 T}{\partial z^2} + \mathrm{CA}(T), \\
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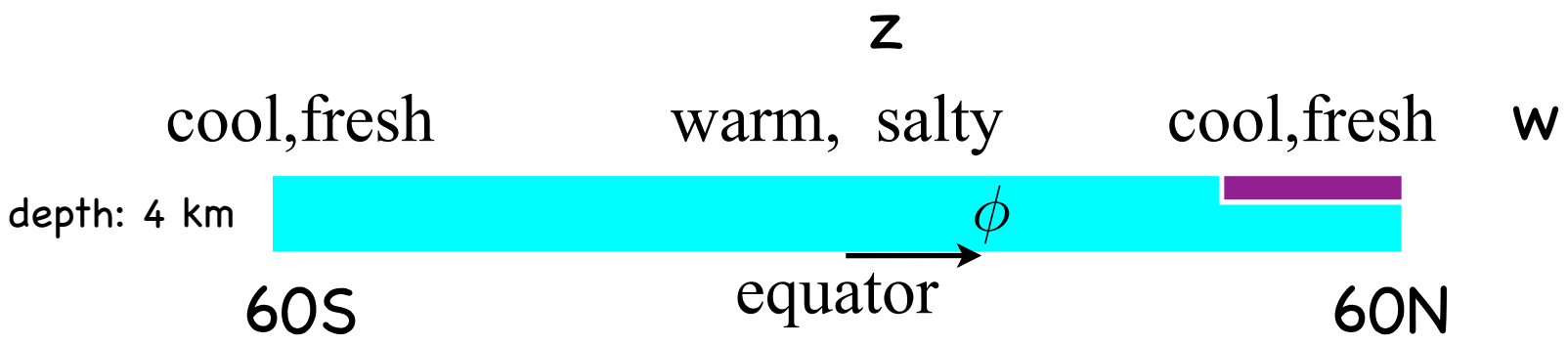
\end{aligned}$$

$1\ Sv = 1$

control parameter: $\beta$

freshwater input northern North Atlantic

# Model Problem

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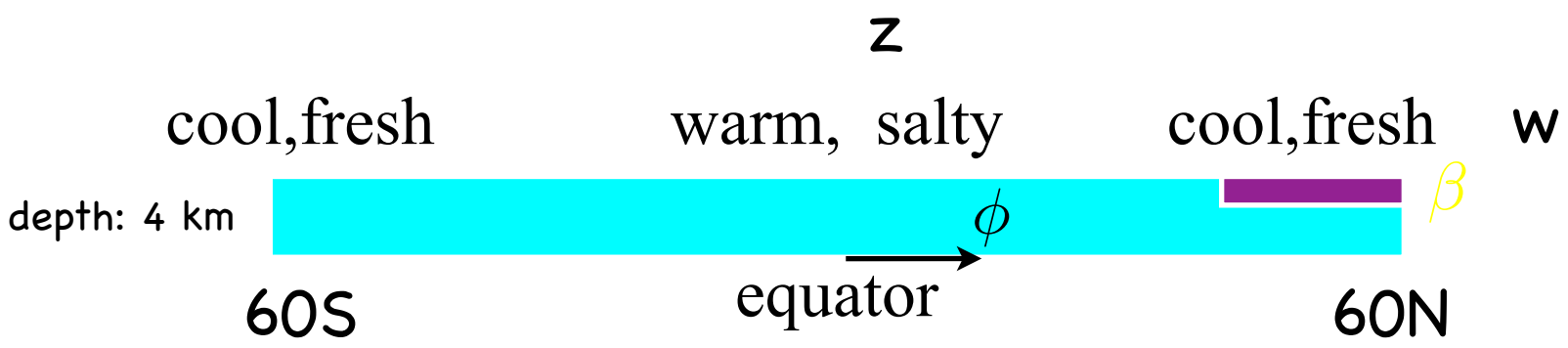


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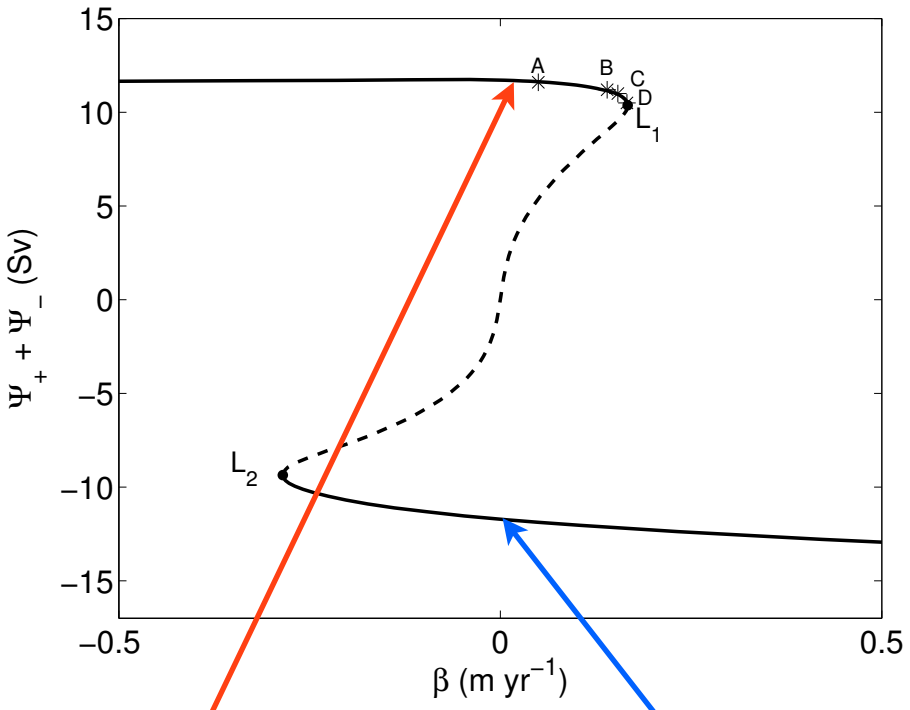
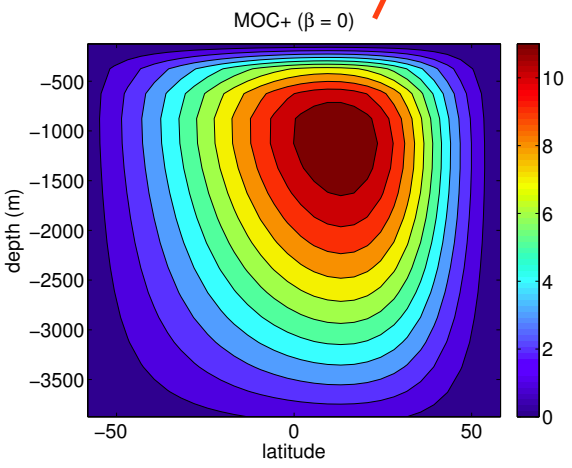
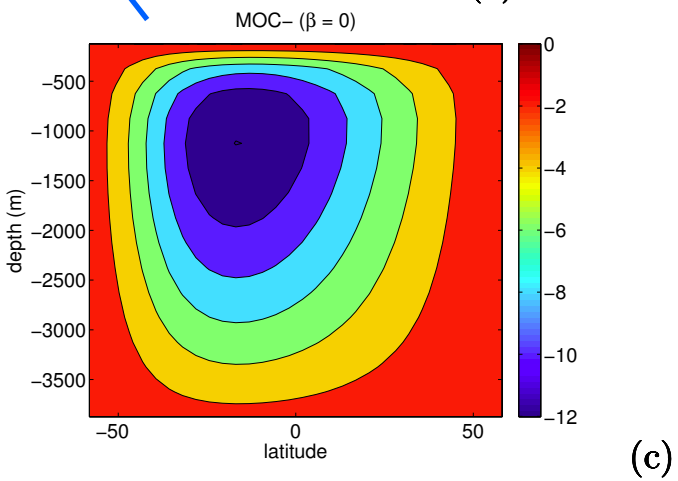
# Bifurcation Diagram

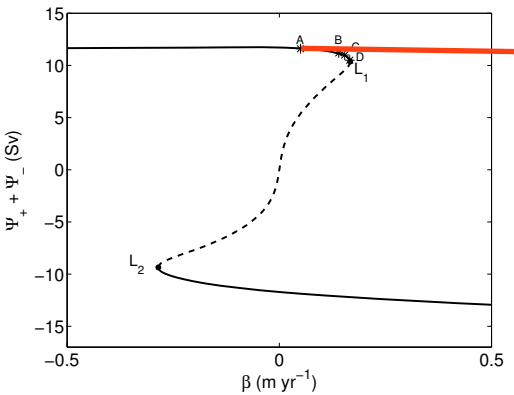


Dijkstra, Nonlinear Physical Oceanography, Springer, (2005)

# Equilibrium Problem

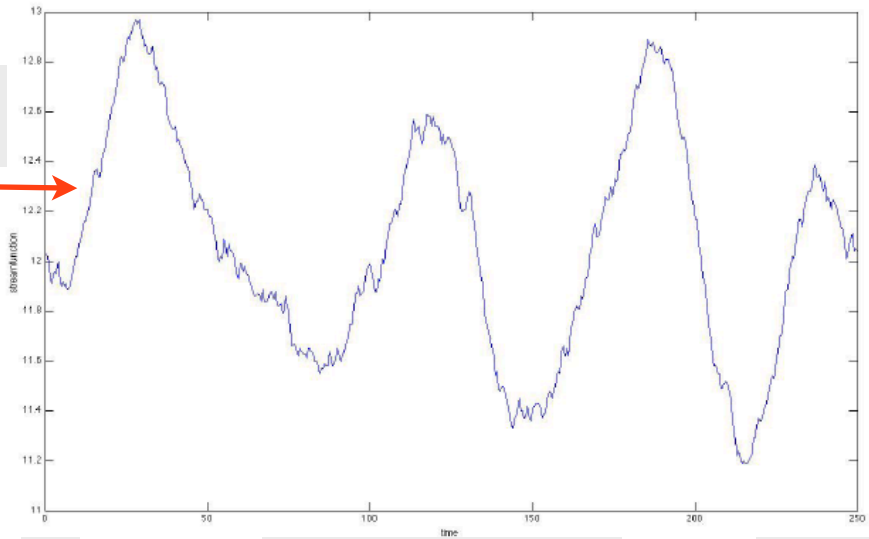



13

MOC

Sv

11



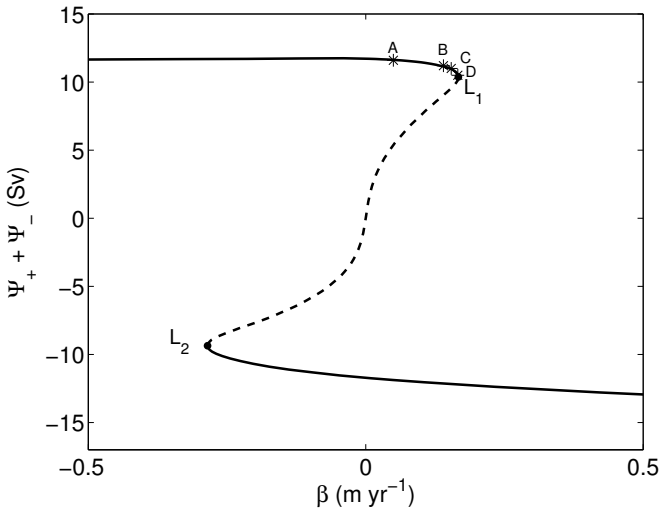

0 time (year) 250

Construct PC network from temperature field
(or salinity field)

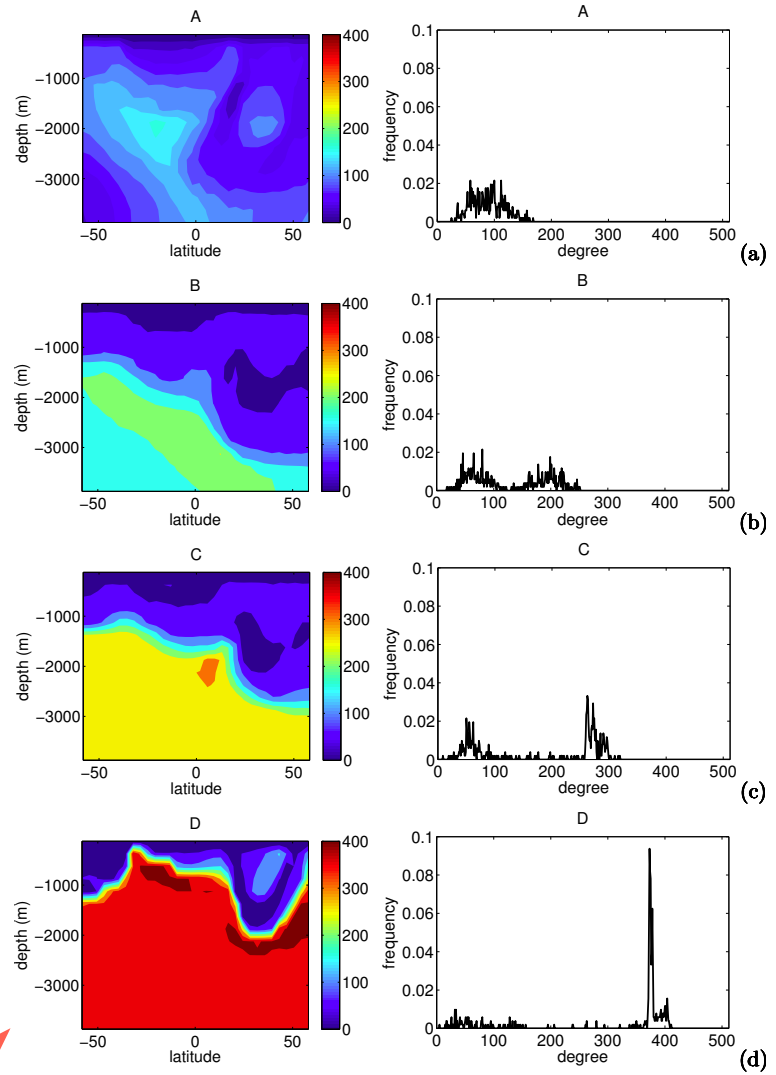
$$\tau = 0.7$$

# Degree distribution changes of reconstructed interaction network from temperature field

grid: 32 x 16

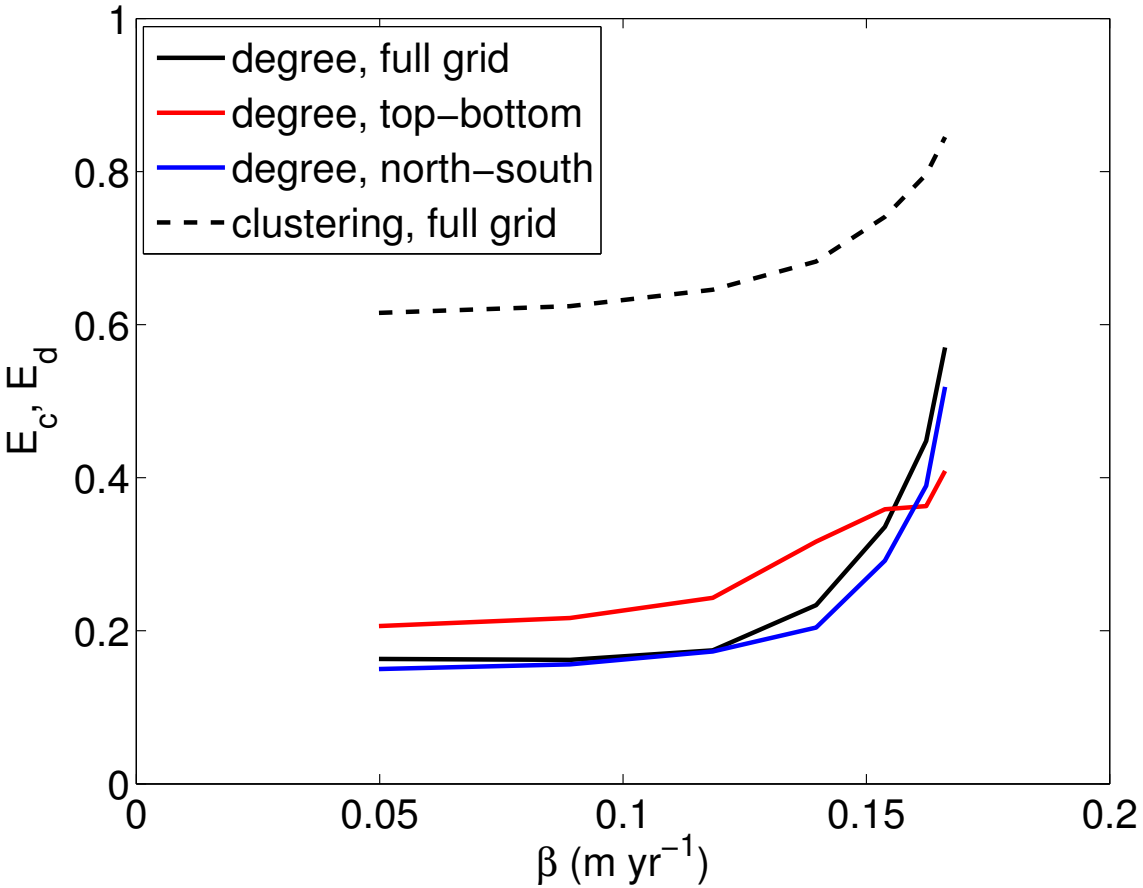


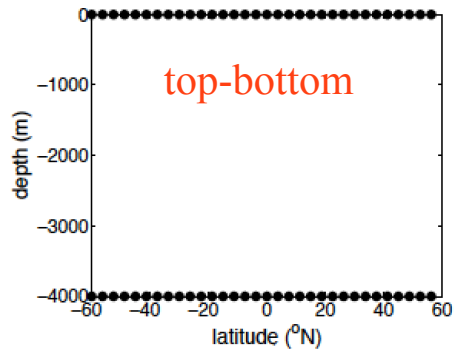

approach to saddle-node

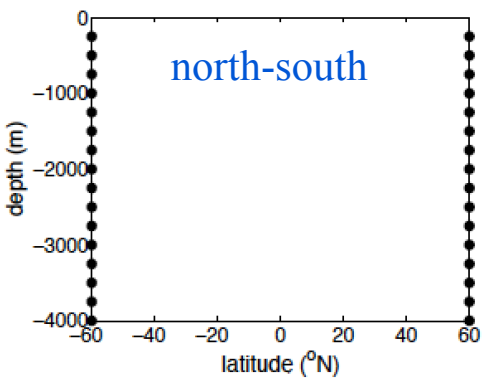



*Spatial pattern of variability becomes dominated by one eigenvector (or EOF)!*

# Interaction network based `Early Warning Indicators'

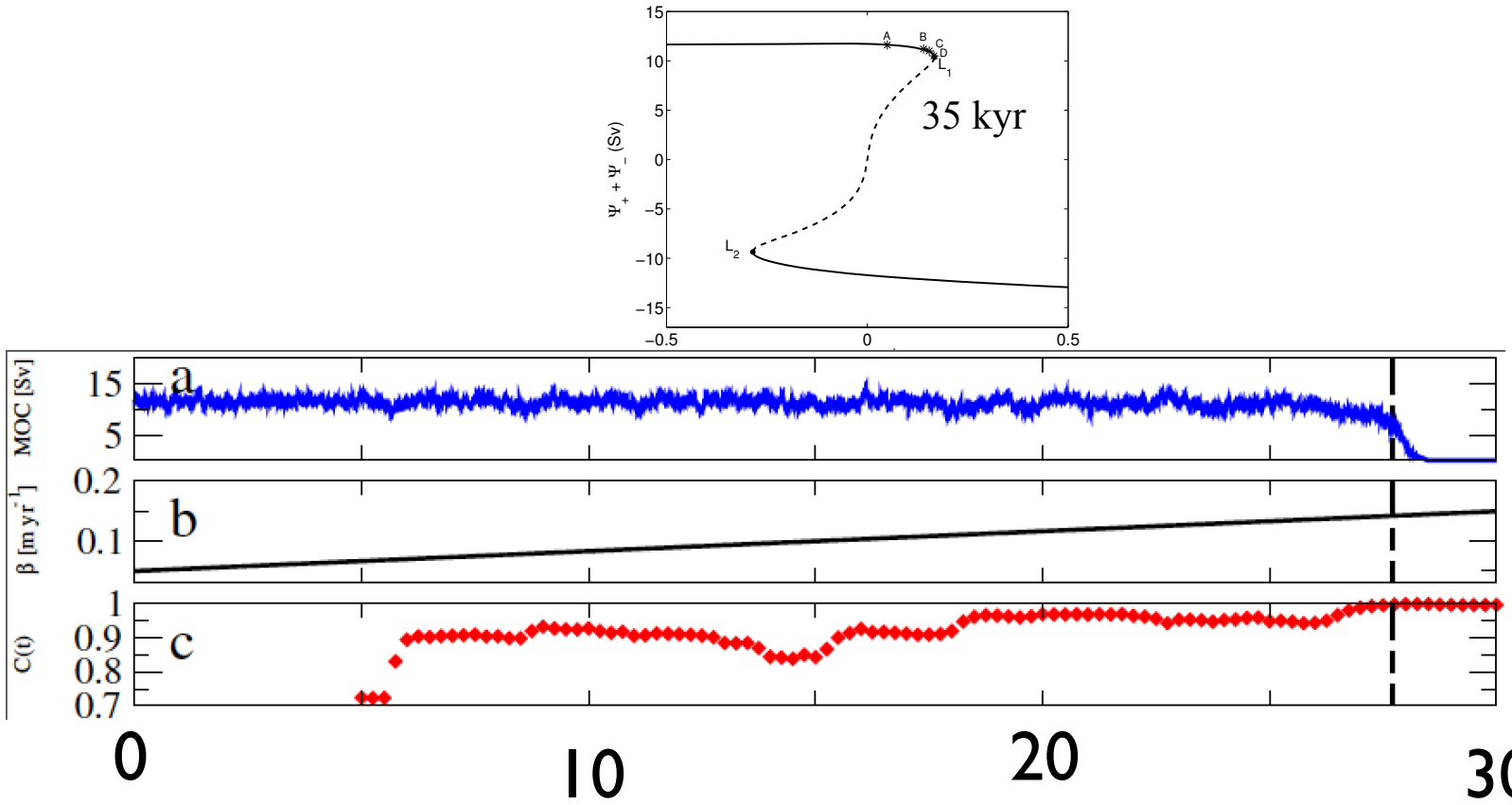



Van der Mheen et al., GRL, (2013)

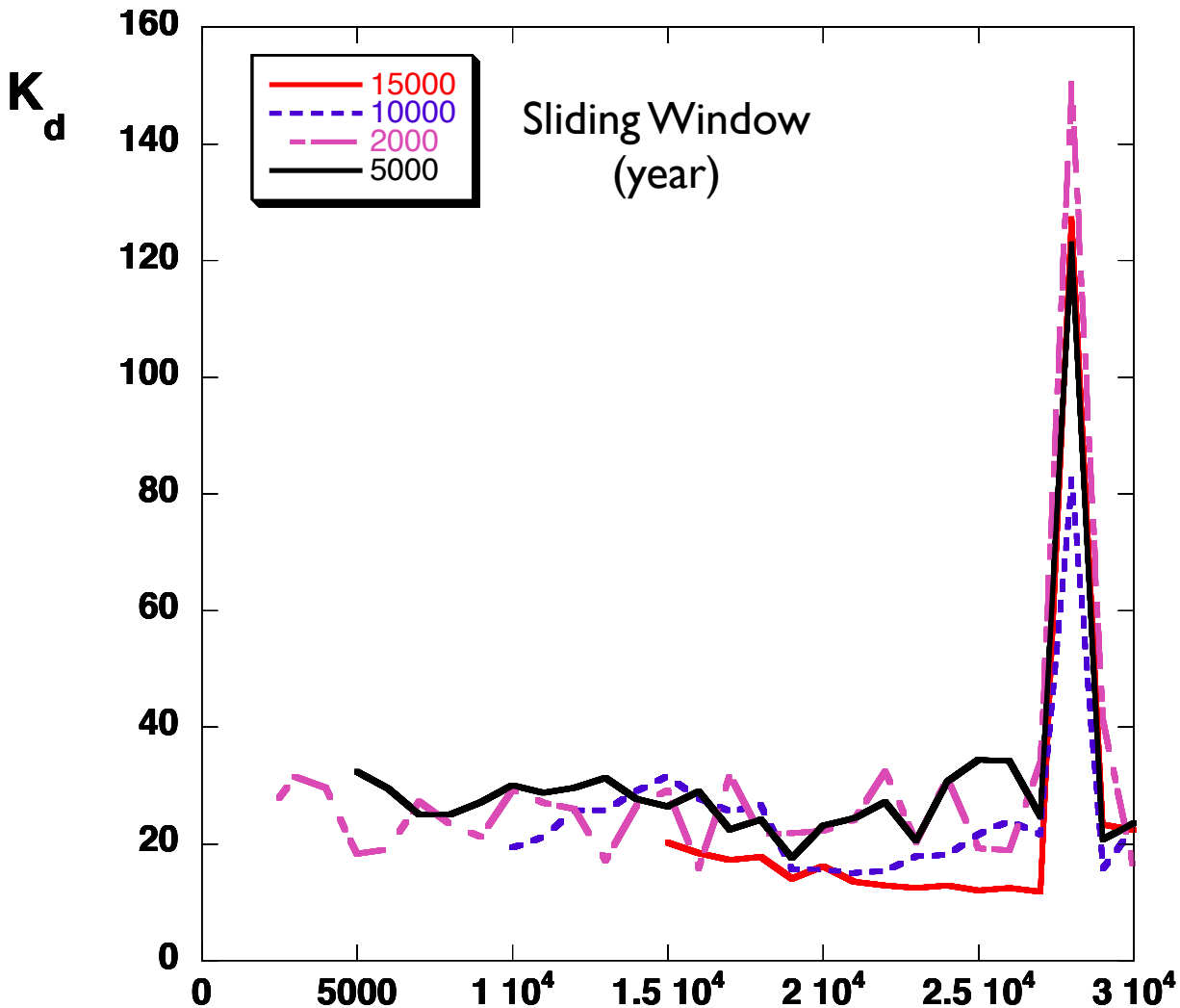
# Transient problem: freshwater forcing is changing slowly in time



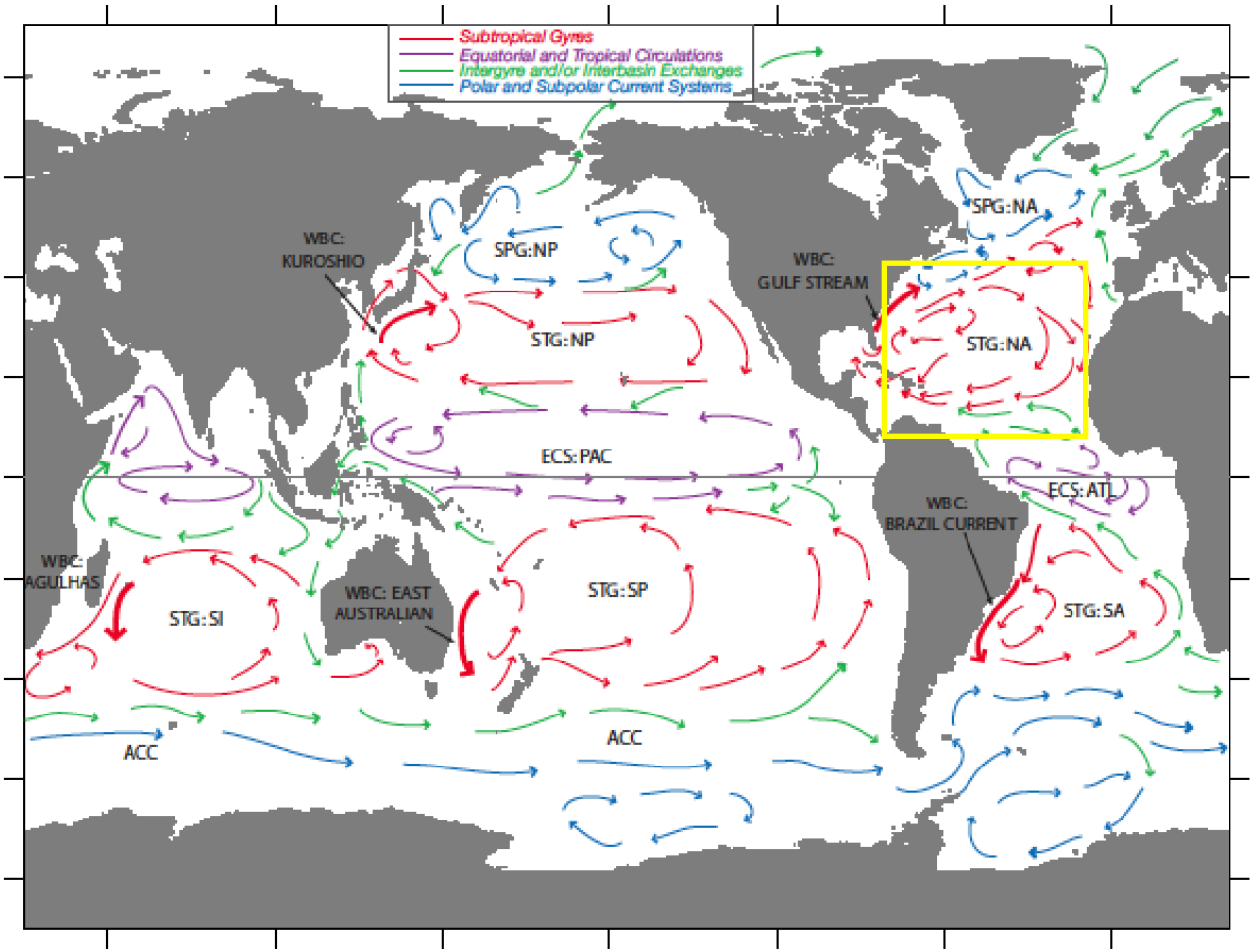

Construct network from (detrended) temperature field (or salinity field)

# Results

## Kurtosis of Degree Distribution



# Application: Flux Network




*Vallis (2006)*

# The Single-Gyre flow

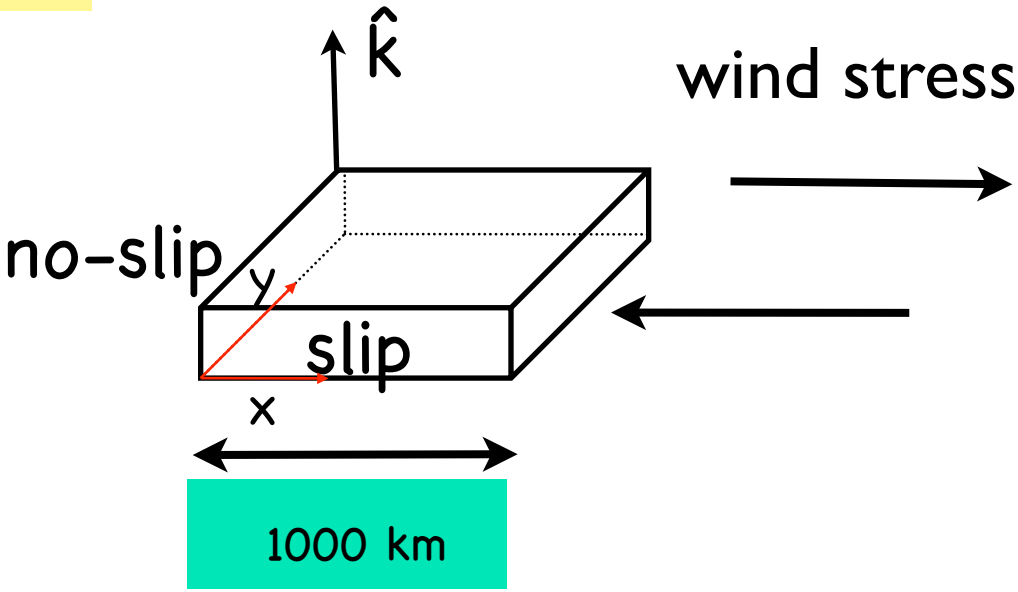
$$\frac{\partial \mathbf{v}}{\partial t} = -\nabla p + \frac{1}{\mathrm{Re}}\nabla^2\mathbf{v} - \mathbf{v}\cdot\nabla\mathbf{v} - f\hat{\mathbf{k}}\times\mathbf{v} + \tau_w(\mathbf{x},t)$$

`friction'  Coriolis  wind-stress forcing

$$\nabla\cdot\mathbf{v} = 0$$

$$\mathrm{Re} = \frac{UL}{A_H}$$

Domain and boundary conditions




**v** = (u, v)

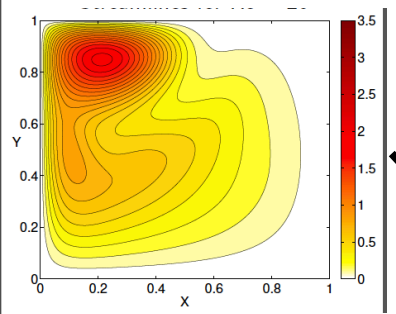
velocity vector

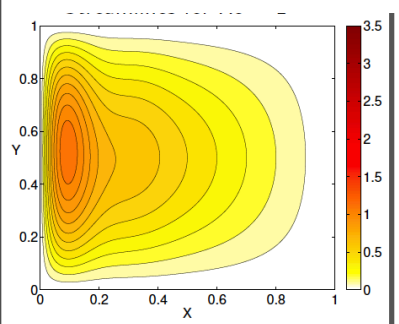
$= (-\partial_y\psi, \partial_x\psi)$

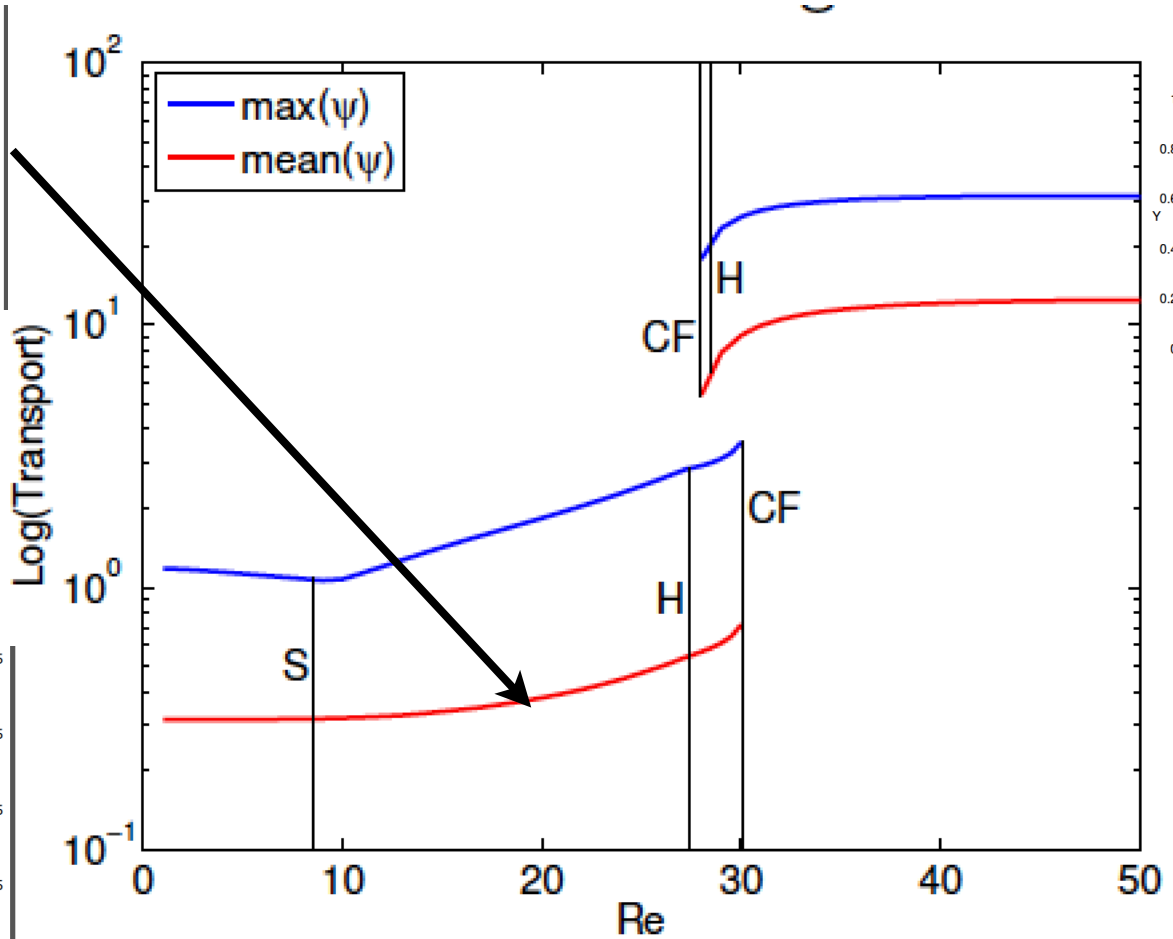
barotropic streamfunction

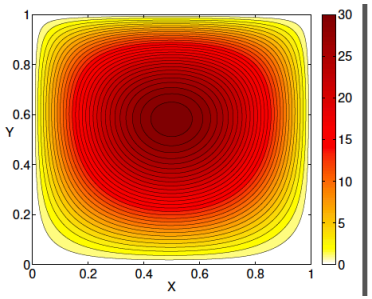
# Bifurcation diagram

## Streamfunction







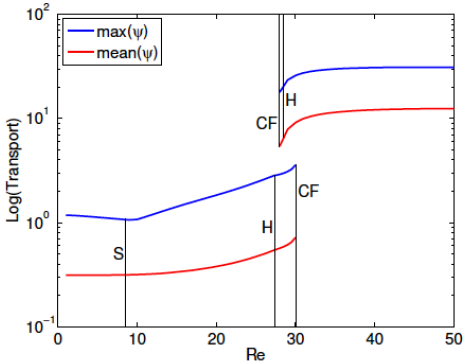
S: Structural bifurcation

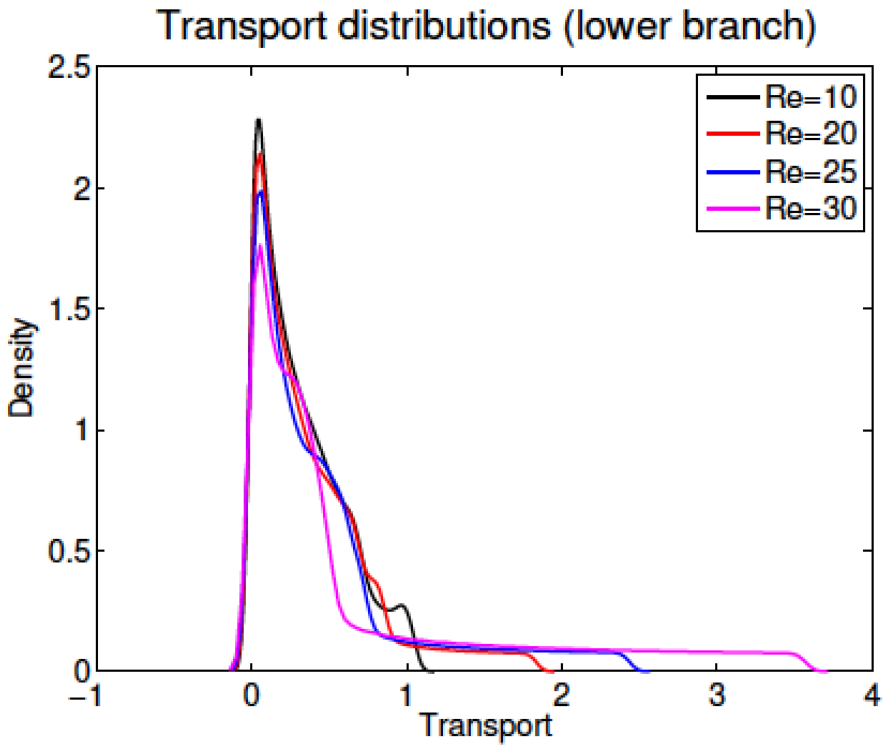
H: Hopf bifurcation

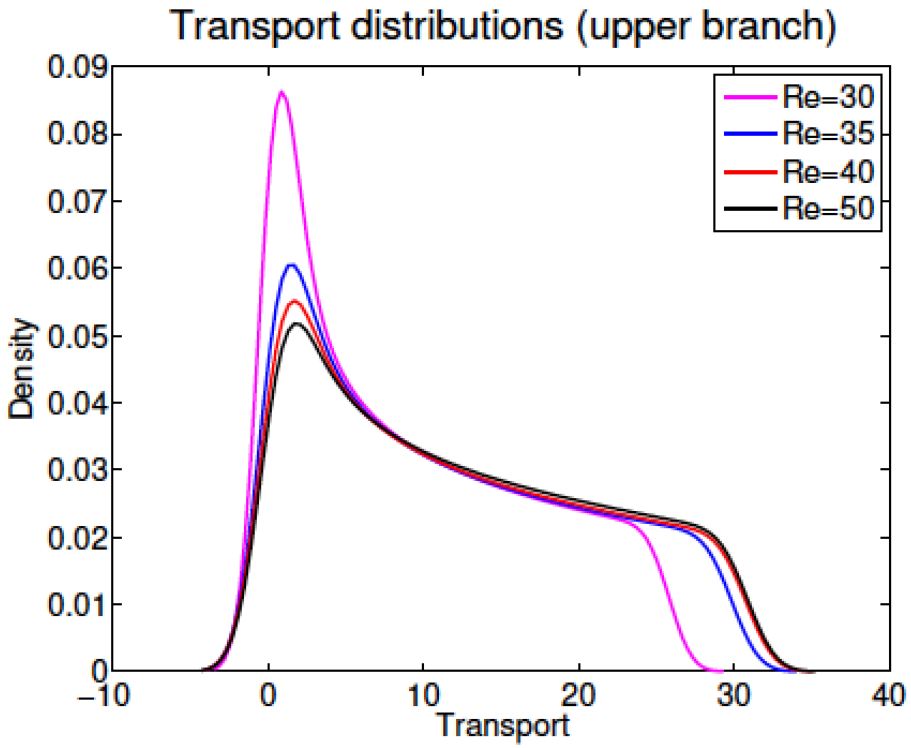
CF: Cyclic fold bifurcation

Dijkstra, Nonlinear Physical Oceanography, Springer, (2005)

# Transport distributions



# Flux Network

$$\begin{aligned}\mathbf{u} &\equiv (u, v) \\ &= (-\partial_y \psi, \partial_x \psi)\end{aligned}$$

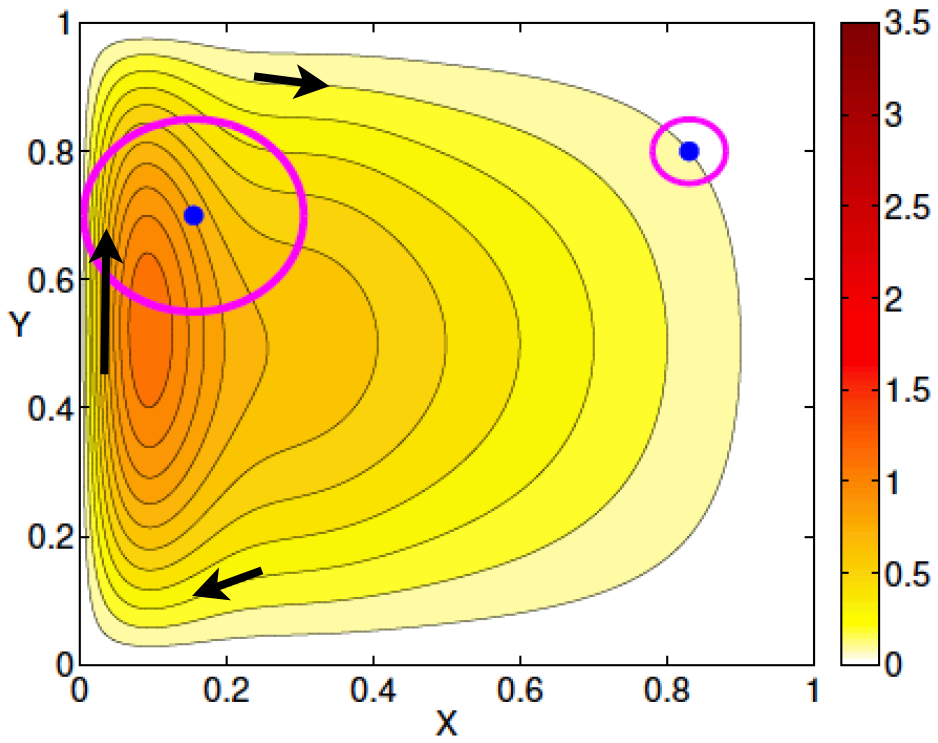
Direction:

$$\underline{\nabla}\psi / |\nabla \psi|$$

Magnitude:

$$\|\mathbf{u}\| = \sqrt{(\partial_x \psi)^2 + (\partial_y \psi)^2}$$

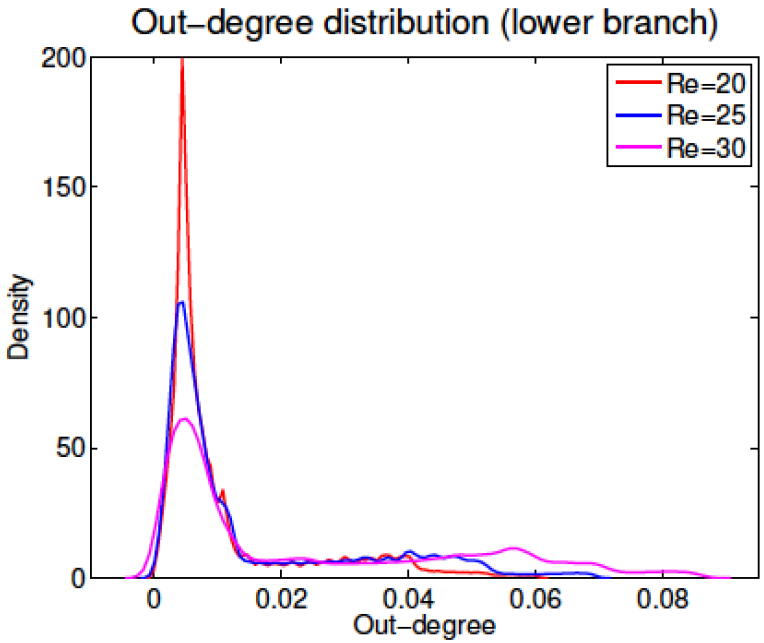
Streamfunction

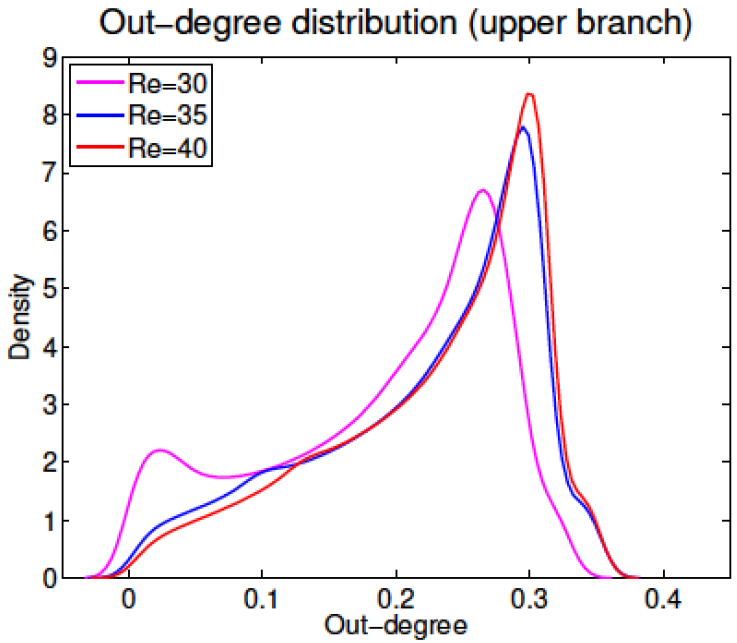


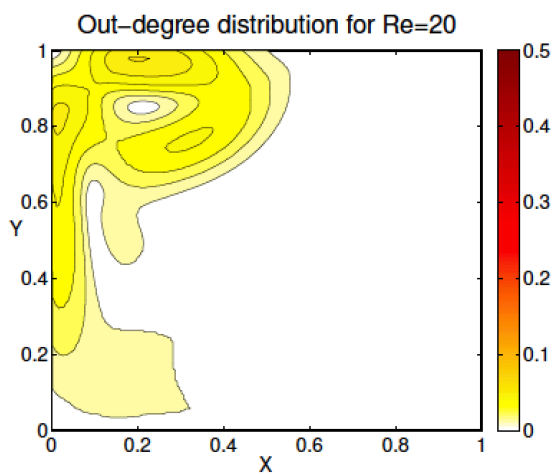

**Local link definition**:

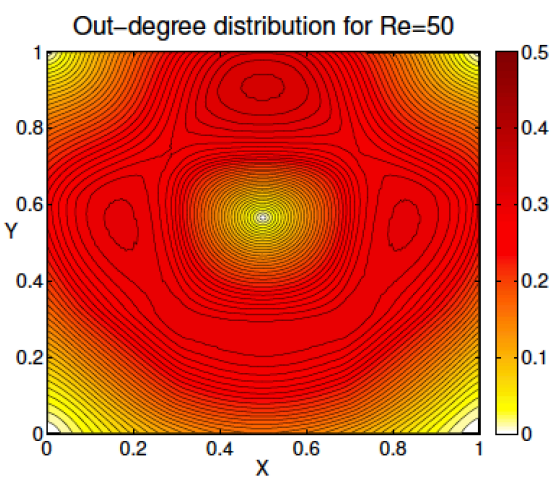
$$N_i = \left\{ n_j \in \mathcal{N} : \|n_i - n_j\| \leq \sqrt{\frac{L}{\pi}\|\mathbf{u}\|} \right\}$$

# Results: out-degree



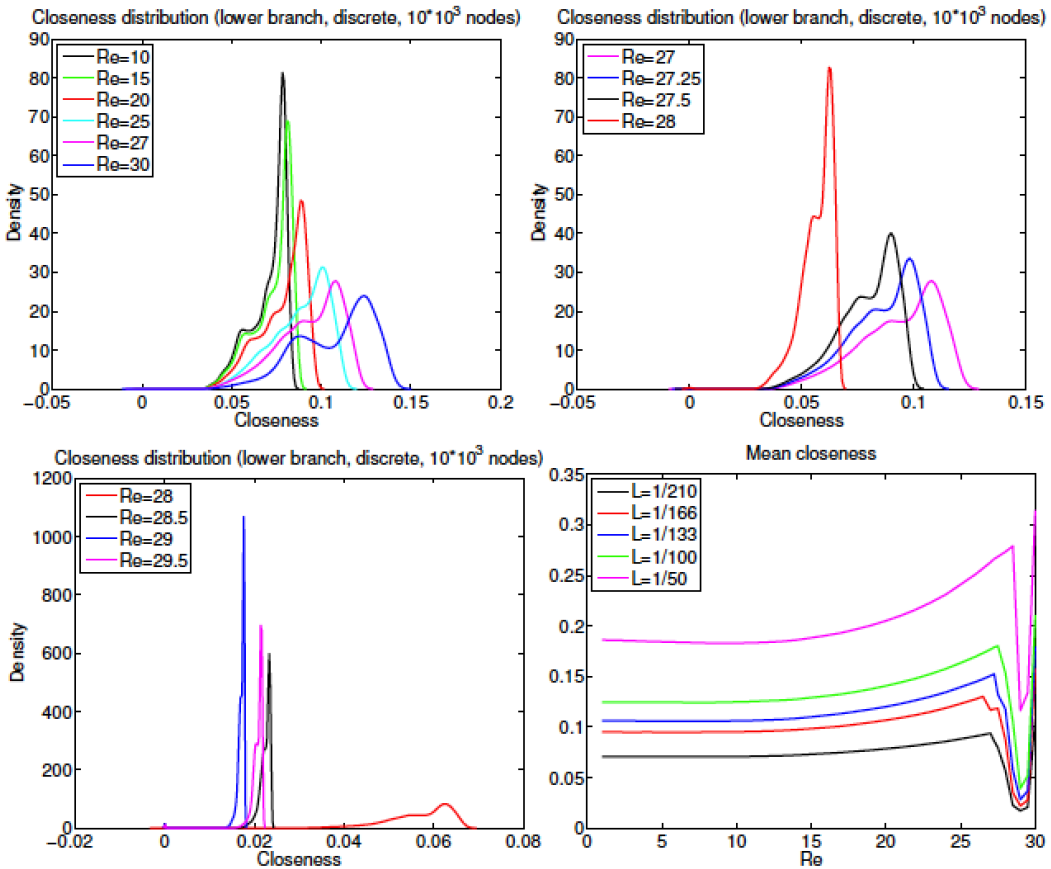



# Leading indicators of qualitative change: short time warning

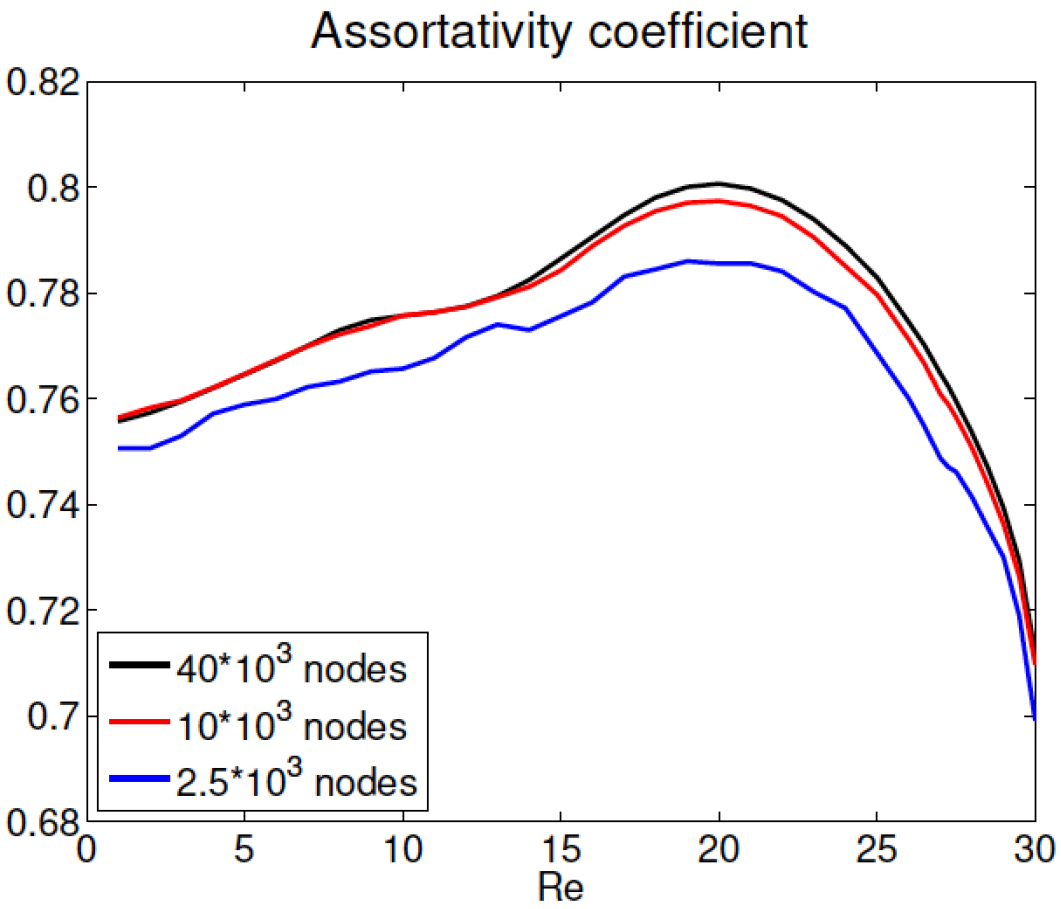
Closeness: measures how easily nodes be reached from a specific node




measures spatial changes in the streamfunction pattern

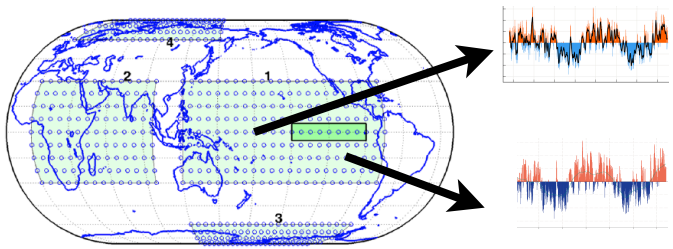
# Leading indicators of qualitative change: long tern warning

Assortativity: measures the extent to which linked nodes are similar in degree



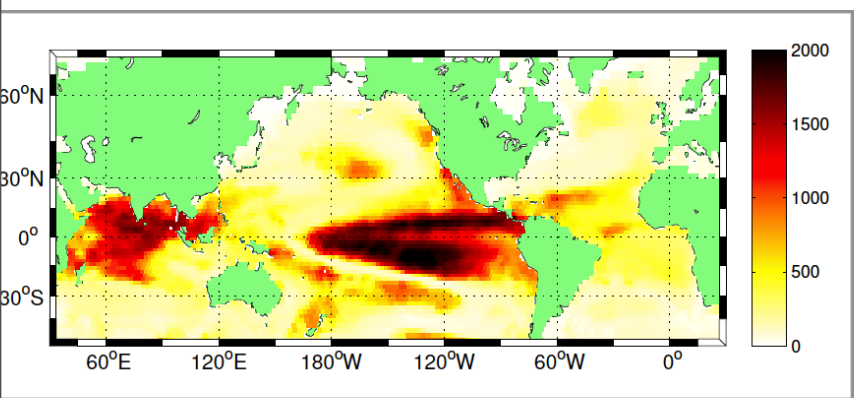

Viebahn & Dijkstra, submitted, IJBC, (2013)

# Summary & Conclusions

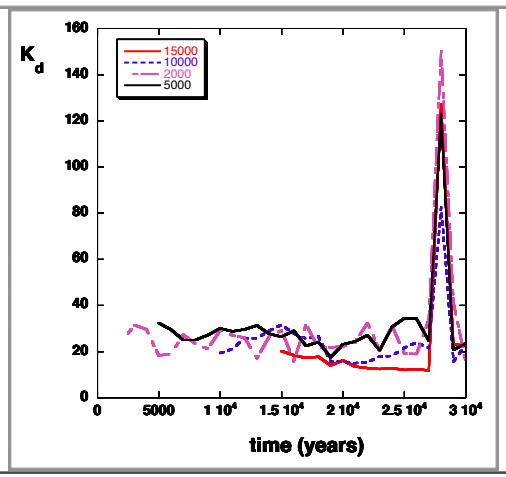


Climate networks can be reconstructed in several different ways

Problem interaction networks:
how does a 'link' connect to the physical processes in the system?



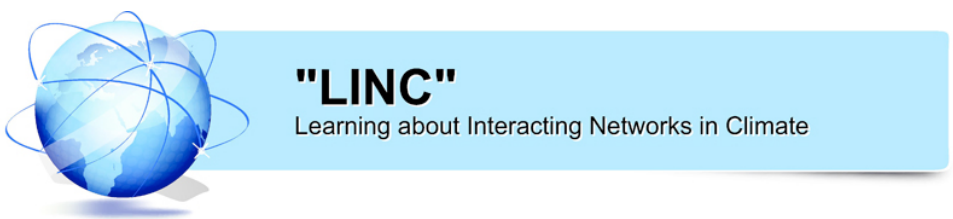

Networks analysis: detailed measure of spatial correlations.




Network based measures can be effective indicators for transitions in spatial patterns

# Thanks!

Mirjam vd Mheen, Matthijs den Toom,
Jan Viebahn, Qingyi Feng, Alexis Tantet
(IMAU-UU)

Juergen Kurths (PIK), Avi Gozolchiani (BIU)
Emilio Hernando-Garcia (UB)

and all other members of