PHYSICS AND ASTRONOMY

College's Day Maths Test

Attempt as many questions as possible within the allotted time. Allowed time 45 minutes. Calculators permitted.

1. Sketch the graph of $y = x^2 + 8x + 19$ beginning at the minimum point, and ending at the *y* axis intercept.

2. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ of $y = e^{2x} - 10e^x + 12x$.

3. If $\frac{dy}{dx} = 3x^{\frac{1}{2}} + \frac{16}{x^2} - 7$, find $\frac{d^2y}{dx^2}$.

4. Solve $\cos\theta(\sin\theta - 3\cos\theta) = 0$ in the interval $0 < \theta < 2\pi$.

5. Given that $\log_a x = 2\log_a 6 - \log_a 3$, show that $x = 12$.

6. Given that $\log_a y + \log_a 5 = 7$, express *y* in terms of *a* giving your answer in a form that does not involve logarithms.

7. Find:
   a. $\int x\sqrt{1+x^2}\,dx$,
   b. $\int e^{3x}\cos(e^{3x})\,dx$.

8. Evaluate $\int_1^2 \frac{x\,dx}{x^2+1}$.

9. $\frac{\partial}{\partial x}$, which may also be written f_x , signifies partial differentiation where a function $f_{(x,y)}$ is differentiated with respect to the stated variable with all other variables held constant.

   Given this, if $f_{(x,y)} = 3x^2 + 4xy - 2y^2$, find;

   a. $f(2,-3)$,

   b. $f_x(2,-3)$,

   c. $f_y(2,-3)$, and

   d. $f_{xx}(2,-3)$.